Mathematics Education -
Relevant, Interesting and Applicable

MERIA PROJECT IMPACT ANALYSIS

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MERIA PROJECT IMPACT ANALYSIS

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Project MERIA, August 2019.
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Executive report and overview

Introduction

The focus of project MERIA is on promoting inquiry based mathematics teaching (IBMT) across four partnering countries (Croatia, Denmark, The Netherlands and Slovenia), on producing innovative teaching and learning materials and supporting teachers in the process of implementing new teaching approaches, all with a goal to investigate what can make mathematics teaching more relevant, interesting and applicable to our students.

It was one of the main aims of MERIA to strengthen the connections between secondary mathematics education and universities, as well as to provide opportunities for professional development of teachers in the field of inquiry based teaching and other student-centred approaches. Project MERIA has been valuable as a meeting point for educational institutes, especially higher educational institutions and secondary schools. It provided an environment for educational experts with various background to collaborate and develop innovative teaching practices. As well, the project team strived to provide additional opportunities for upgrading teaching skills of in-service mathematics teachers. Accordingly, project MERIA tried to promote the profession of mathematics teachers and highlight the importance of high quality teachers for mathematics education.

During the project's lifetime the project team has produced several intellectual outputs to facilitate its goals. First is the *MERIA Practical Guide to Inquiry Based Mathematics Teaching* in which the basic tenets of theoretical frameworks are described and illustrated with many examples. The Practical Guide was written for teachers by researchers in mathematics education following the well-known frameworks of Theory of Didactical Situations and Realistic Mathematics Educations. Second output is the *MERIA template for scenarios and modules* which presents the structure for the teaching materials developed later on in the project, together with one exemplary scenario and module. One of the main results of the project is a compilation of five *MERIA scenarios and modules* that have been tested and developed in cooperation with teachers in associated schools in all four countries. The scenarios are showcases of didactical situations that provide a very important insight and support for teachers trying to implement inquiry based mathematics teachers in their own classrooms. Finally, as the change of teaching methods may be quite challenging, we have designed a cycle of professional development courses, with all activities described in detail in the *MERIA Workshop Guide*. These workshops have already been delivered to around 100 teachers in the four partnering countries. All of these outputs have been evaluated and improved to support the MERIA team and the final users (mathematics teachers) in achieving the desired goal.

Along with online versions available with free access at the project's official webpage, *MERIA Practical Guide* was published in Croatian financed by the

Croatian Mathematical Society. Publishing of *MERIA scenarios and modules* in Danish has been financed by the Ministry of children and education in Denmark, with a shorter introduction to the theoretical framework prepared by the project team based on the *MERIA Practical Guide*. Both publications are aimed to be distributed to mathematics teachers nationwide.

Quantitively, all targets set out in the application form have been achieved. In total, working with associated schools and through workshops in all four countries, we have introduced MERIA and retrained around 150 teachers. Among these, 31 teachers from 13 associated schools have participated in semi-structural interviews. During the design phase in 2017 and 2018, teachers in associated schools have tested eleven MERIA scenarios in 64 lessons and 1290 students have provided feedback on the relevancy of the lesson and their satisfaction. Furthermore, almost all teachers participating in the MERIA professional development cycle were asked to implement at least one scenario and report about the outcome, so we estimate that about 80 more lessons have been performed based on MERIA materials with around 2500 students participating. In general, the teachers reported that the structure of the scenarios was helpful to implement IBMT and the results of the student questionnaires show that we succeeded in particular in creating interesting mathematics lessons for them. Using national professional networks of teachers to send out newsletters and to communicate our experiences, MERIA was promoted among all secondary mathematics teachers in Croatia, Denmark, the Netherlands and Slovenia, and the official MERIA Facebook page has gathered a community of 375 followers.

This booklet comprises of various (research and data-collection) tools and reports that have been developed to measure the project's impact and orient the project team in the design of the results. *MERIA Project Impact Analysis* has two goals: first, to analyse the use of different teaching and learning methods, especially IBMT elements, in secondary schools mathematics classes in participating countries in order to compare the situation, to find out best practices and to design interventions (materials and workshops); and second, to evaluate the project impact on school level (in the associated schools) and teachers participating in different project's programmes and activities.

The comparison of educational systems in partnering countries was based on the relevant document analysis and it is organized into main domains similarly to those for semi-structured interviews of teachers. Thus, the organization enables subsequent comparison and matching of regulations, recommendations and our expectations to the existing teachers' actual practices in the schools associated to the project.

Comparison of educational systems in partnering countries

A substantial document analysis has provided valuable information on the state of the art in mathematics education in partnering countries concerning the main aspects relevant for the design of project results. Curricula in all partnering

countries prescribe content to be covered by certain programmes and bring recommendations or guidelines on how topics could be presented, but with no strict rules concerning teaching methods, although students' active learning strategies are officially promoted. Comparing the topics listed in the national curricula of partnering counties, we arrived at two categories of content areas. First category served as a source of topics that will easily fit in programmes of all partnering countries, while the second category provides ideas for additional (extracurricular) modules. In addition, we bring a description of mathematical processes that are encouraged to be used by some national curricula.

Evidence from international studies PISA, TIMSS and TALIS point out that traditional transmissive practices predominate in everyday classes in many countries, which might act as a constraint for the implementation of IBMT. Students in partnering countries show various achievements in PISA testing. While scores in mathematical literacy for the Netherlands, Denmark and Slovenia are very similar, and in the Europe's sub-top (scores between 510 and 512), Croatian students perform significantly lower (score 464, while the OECD score is 490). Low achievement in mathematics education may be addressed by innovative teaching methods, popularization activities and other activities (e.g. participation in projects and different forms of competitions) that increase students' motivation. Surveys show that innovative teaching methods, e.g. inquiry-based mathematics teaching (IBMT) increases students' interest for mathematics.

Regarding textbooks, national policies vary concerning their approval by the national authorities, but in each partnering country a great majority of teachers follows one or two main textbook series. For preparing a lesson, teachers also use other resources. Except in the Netherlands, materials written to be used by inquiry-based approach are scarce and hardly available. This may indicate that there is a gap that could be filled by new materials designed in project MERIA providing teachers support in implementing IBMT. It is a requirement for these materials to be developed according to standards set by the most used textbooks. In some countries, teachers autonomously write their lesson plans which usually include information on learning goals for the lesson, learning methods, student activities and didactical material, but this could not have been regarded as a rule.

There is a variety of opportunities for professional development in partnering countries. The topics of meetings vary and do not follow a strict programme, neither do they focus on a specific theme. The subject of inquiry based teaching has been present in workshops for professional development, but practical information or "hands-on experience" supporting teachers to implement such approaches on their own is rare. MERIA workshops focused on the practical side of using IBMT approaches and provided teachers with materials and guidance in doing so. Consequently, MERIA workshops have some overlap with existing professional development, but nonetheless this kind of workshops provided a new and valuable experience for teachers in all partnering countries.

Assessment regulations in all partnering countries have been viewed in their major aspects: the role of external summative assessment in the form of the graduation (final) exam, the role of assessment in school, and the extent to which formative assessment is regulated. For MERIA, we derived two important conclusions. First, the project team was aware of the strong barrier to implementation of IBMT - limited time that teachers are willing to "sacrifice" for skills not tested in the external final summative exam. Second, the need for evidence of students' progress during "research-like" activities was discussed. The project tried (along with new materials) to provide teachers with formative assessment methods that would give them and their students feedback on student's progress in attaining the target knowledge expected in the scenarios.

Analysis of semi-structural interviews at the beginning of the project

The MERIA team chose to investigate the difficulties and challenges that teachers face for implementing IBMT. In associated schools, 31 teachers participated in semi-structured interviews to express the problems they are confronted with in their daily practices, and their opinions on the changes needed to support learners in their classrooms. The schools were chosen among similar programmes in each country, having 3-4 hours of mathematics per week. It was noted that teachers in different school programmes with mostly heterogeneous classrooms, meaning with students with different pre-knowledge, are in special need for appropriate support in implementation of IBMT. In addition, teachers gave a whole range of useful examples of good practices. Finally, the interviews provide valuable information for the project team to streamline further project activities.

Based on the findings in the analysis of first round of interviews, we concluded that the interviewed teachers share an interest in inquiry-based approach and a positive attitude towards making changes in their teaching. Their familiarity with IBMT is not very deep and they would welcome instructions on implementation of alternative lessons and scenarios. We also conclude that teachers do not think in terms of a specific educational theory and do not implement IBMT, although in all countries teachers occasionally introduce some elements related to the problem-based and inquiry-based learning. Furthermore, we tried to stress that implementing IBMT does not imply that what they do is not effective, but that this IBMT enriches their teaching practice and helps them to better support their students by making mathematics more relevant and interesting.

Teachers assume that there are benefits of the IBMT approach, but they mostly recognize challenges. Based on their answers, the main obstacles to applying IBMT are lack of time to prepare (innovative) lessons, lack of students' motivation for learning mathematics, lack of quality materials supporting IBMT and the general orientation on final exams (assessment) with routine tasks that is required by the system. In addition, teachers are sceptical about the classroom and time management, so they would be more confident to use new materials only if they are well prepared. We may say that the interviewed teachers are very enthusiastic. This is not a surprise since they agreed to participate in the project,

but we should be aware that their level of motivation is above the average teachers. For the project this meant that in teachers we have interested partners, who seek additional education in IBMT. We also noted that for teachers to accept a new approach, we needed to convince them that it is worth their (in-classroom and preparation) time.

Concerning systematic changes, answers indicated that teachers in different countries have somewhat different problems and concerns. In Denmark many questions were hard to answer at the time because of the curricular reform. In the Netherlands, teachers emphasized they would like more freedom in organization of teaching. In Croatia on the other hand, teachers seek modernization of teaching approaches and support in using contemporary methods. They also found that the curriculum and the teaching methods are not well aligned between elementary and high school. In Slovenia, teachers would like to reduce the number of students in a class and the number of subjects.

Concerning learning outcomes, it was hard to gather, analyse and compare results in different countries due to different use of terminology. Similar concepts of learning goals and target knowledge are also widely used. It is apparent from the interviews that teachers do think about what their students need and will know at the end of the cycle, but it is not clear to which extent the constructive alignment is achieved (in preparation, teaching and assessment). MERIA Scenarios and Modules use the notion of Target knowledge and Broader goals in the scenario, while the rationale behind the scenario is explained in the module. Module also contains short exercises designed to test if the target knowledge has been achieved. Finally, these issues were addressed during workshops (and hence described in the *MERIA Workshop Guide*).

The issue of assessment in general has been addressed by one hypothesis and one research question. Interpretation of the results is delicate. We conclude that the alignment of assessment and the teaching approach is achieved only when teachers use traditional methods (frontal approach combined with classroom discussions, a lot of practice and written exams). Assessment in general is one of the greatest challenges for the teachers and they often mentioned that they would appreciate further training. The aim of the project MERIA was to promote IBMT, knowing that it requires long time for students and teachers to adapt to the change, so emphasizing that in the beginning we must encourage interest, engagement and free interaction in the classrooms and there should be no pressure in terms of assessment as that would ruin the potential for learning. In that respect, we have set a goal for the project to provide more ideas for formative assessment and achievement of constructive alignment throughout MERIA workshops.

The supply of teacher training is abundant in all countries, but the quality of the offer varies. Teachers find some workshops very useful, while some are, in their opinion, a waste of time. Only singular topics or events are mentioned and there is no mention of any form of systematic continuous professional development.

Our interpretation is that longer overarching programmes of in-service teacher training are missing. Meetings with teachers in associated schools were hence more frequent than it was planned in the project application. MERIA workshops were not organized as disconnected events, but as a series of workshops with opportunities for teachers to try out and reflect on the material in between. We hope that the workshops organized in the final stage of the project are only a starting point of a longer programme of continuous professional development on the topic of IBMT. Our goal was to make basic concepts of IBMT precise, discuss drawbacks and benefits supported by instructive examples and evidence, and provide useful tips on classroom management to address teachers' concern about time.

Analysis of semi-structural interviews at the end of the project

At the end of the project, in July 2019, we have repeated the interviews with 15 teachers from associated schools in Croatia, the Netherlands and Slovenia. These teachers participated in meetings with project members during 2018 as project's methodology and results were under development. Nonetheless, all of them first participated in lectures on basic tenets of IBMT, TDS and RME, then experienced at least one MERIA scenario and discussed the implementation with others at the final meeting of the cycle. Eleven teachers participated in the Final Conference. All of the interviewed teachers are familiar with the Practical Guide, but have not seen MERIA modules as these were in part written based on their experience. It is clear from the interviews that teachers evaluate MERIA scenarios as detailed and useable and are satisfied with students' reactions.

Teachers appreciate that they have received detailed materials and an opportunity to discuss the implementation with other colleagues. From the materials they value the structure as it provides them the way how to implement inquiry based teaching, while feedback from other colleagues helps them to gauge their expectations and improve their actions. Teachers' understanding of inquiry based teaching is mostly experiential and they define it as "students working on their own". This understanding has certainly evolved through workshops as they report that their perspective has changed, and they provide examples on it. Concerning their own skills, teachers have improved in their role to "withdraw in a meaningful way" and value the time given to students to work without their interference.

Teachers claim that they would like to share their knowledge with other colleagues and introduce them to inquiry based teaching, but it is our impression that it might be still very difficult for teachers at this stage of the project to move on from their role as teachers to teacher trainers. On the other hand, teachers clearly illustrate their values and support their reasoning with positive examples. Teachers have not adopted the vocabulary of the presented frameworks, but it is clear from the interviews that they use the concepts that are central to these frameworks. For example, teachers emphasize students' autonomous actions ("adidactical situations"), they often mention students' expectations and time needed to adapt to a novel way of learning ("didactical contract"), they are aware

of the structure of a lesson ("phases in TDS") and of their role to support students ("scaffolding") and to connect students work to the topic of the lesson ("target knowledge" and "institutionalization"), also they value realistic and open problems ("rich contexts in RME" and "didactical potential"). As a form of professional development, they rate MERIA workshops (and the project in general) very positively and emphasize that they highly appreciate being active participants and discussions about their experiences. They point out that there are a few such opportunities as mostly professional development is provided in a form of lecture with no materials.

We have teachers from different types of schools. Some teachers identify their students as "not so good as in other schools" and they find students' lack of prerequisite knowledge as an obstacle to advance to further topics and experimenting with different types of teaching. Some students show negative attitudes towards mathematics and persisting resistance to engage in a discussion or to think about the problems autonomously. Teachers report that students "learn for grade" and describe students' expectations as "show me the procedure, give me another similar task and grade me". Also, all teachers reported that during the first implementation of a MERIA scenario students were confused by the change of the teaching method and asked for help what to do. Those teachers that tried more scenarios in the same class report that students adapt successively with time and that there was a high level of satisfaction of both students and the teacher. Teachers claim that their teaching is heavily constrained by the final national external assessment ("matura") and the amount of the mathematical curriculum it covers, but they clearly point out that logical thinking is also required at these exams and that MERIA scenarios support its development. So, it is useful to incorporate inquiry based teaching since it supports mathematical reasoning required in the final assessment, but this argument is not always clear to students and their parents. This all points that the negotiation of the didactical contract is crucial for successful implementation of inquiry based teaching.

Almost all teachers discuss students' work in terms of successful solving of tasks ("on the good path", "lost", "did what I expected" etc.) as though there is only one way of solving the problem. Teachers express that they value different approaches and that they are encouraging students to make mistakes and learn from them, but they know with which achievements they are satisfied (which is not necessarily the ultimate target knowledge as written in the scenario) or when they are positively surprised. It seems that teachers have an understanding of benefits of inquiry based approach and a clear set of personal values concerning education.

Teachers feel that inquiry situations bring more dynamics to their classrooms and that MERIA scenarios are interesting to their students. Teachers emphasize the attitudes of students and parents as an obstacle to introduce more changes. They suggest that parents would think that students were only playing and had not learned anything, while they think students work more during inquiry based lessons than while passively copying solutions of tasks. Students get more

engaged because of an element of mystery and it was very positive for the teacher that the students realized that behind a guessing game there is no magic, but a mathematical (scientific, logical) explanation. In particular, students like new ways to present their results (e.g. in the form of a letter to a mayor, or when multiple students write on the board at the same time) and working in a relevant context (discuss payrolls, provide advice, construct a slide). Overall, all teachers report that students were engaged and that they were positively surprised by the students with more modest achievements. Some teachers are planning to incorporate MERIA scenarios and their own similar inquiry lesson in their yearly plan. In general, teachers feel that they have enough time and freedom to incorporate inquiry based lessons during the year and add that this statement is related to the fact they have tried the scenarios ("now that I know what it is about") and received positive feedback ("my students managed and were satisfied"). So, we conclude that it is personal experience that leads teachers' decisions. It is also important to note that teachers express positive attitudes towards the materials because they trust the authors (the authority of the project team). Hence, we further hypothesize that the use of materials and the success of the professional development also depends highly on the persons providing it.

Evaluation of MERIA scenarios

MERIA scenarios were evaluated by the teachers and students in the associated schools. In the first round, answers were gathered after the implementation of the pilot scenario Area enlargement in all four countries. This facilitated the project team to develop guidelines for development of other scenarios. In the second round, there were 11 different scenarios in total and we used the results together with feedback from the teachers during workshops to decide which five scenarios to finalize. In both rounds, teachers and students completed questionnaires using Google Forms after the lesson. The data has been processed quantitatively also using Google Forms.

There were 19 responses in the first round for the scenario Area enlargement: 8 from Croatia, 8 from Slovenia, 2 from Denmark and 1 from the Netherlands. These responses were used to refine the questionnaire, to prepare the project team for the workshop discussion with the teachers and to write the MERIA template for scenarios and modules. In the end, materials and responses from the teachers were used to prepare the MERIA modules and their comments were incorporated in the section describing the possible variations and the didactical variables. Examples of teachers' suggestions are to reduce/increase the time in a certain phase, to change the motivational examples, to use ICT etc.

In the second round, there were altogether 51 responses from 31 teachers (16 responses in Croatia, 20 in Slovenia, 13 in the Netherlands and 2 in Denmark). Teachers were introduced to scenarios by the project team and in almost all cases they had no difficulty to understand the main goals of the scenario. We had a few very valuable remarks and discussions which led to improvements of the scenarios Slide and Spreading the rumour.

According to answers, the teachers find the scenarios well prepared (average grade above 4 out of 5). Issues faced are related to the time needed to prepare implementation of a scenario designed by someone else or to implement a scenario with a topic that is not being covered at the moment (not fitting with the flow of the yearly plan). Time frame is the most discussed didactical variable. We have tried to analyse and incorporate teachers' complaints in the subsequent versions of the scenarios.

Almost all tasks were suitable in the classes in which the scenarios were implemented, but we were convinced that the scenarios Conflict lines – Parabola, Achilles and the turtle and the Trigonometry doubling formula were much more abstract than the others and too hard for some groups of students. Answers to provide further ideas were short, usually with no suggestion from the teachers. This might also indicate that teachers are more interested in the implementation than the analysis or task design.

The teachers in Croatia and Slovenia report that their students were more engaged than during the regular classes, and in the Netherlands the engagement was reported at the same level. This is in line with the answers of students on their perception how hard they worked.

In answers to questions about the flow of the lesson and way they know that students' have understood something, teachers mostly describe what the students were doing in each phase. Their main cue for students' understanding seems to be the "aha!" response. Some teachers report that it is hard for them to answer these questions. This was also discussed with the teachers in person. The impression of the project team was also that teachers find it much easier to discuss students' work than their own actions. On the other hand, reflecting on these questions is an important part of professional development for the teachers changing their style of teaching. Overall, teachers show satisfaction with the lessons, in particular for the five scenarios we have in the end chosen to publish.

Questionnaire for students provided a lot of data on a reasonably big sample of student population, but it must be emphasized that the research was much more oriented on probing students' reflexion on the lesson to understand their interests than to validate the methodology behind the design of the scenarios. Hence, we do not claim any significance of the conclusions that we present, but we explain which data we considered most important for further actions.

We should note that the sample is considerably smaller in Denmark (34 students) as opposed to other three countries (189 in the Netherlands, 306 in Slovenia and 389 in Croatia). Answers show that the students find MERIA scenarios clear, well timed, not too difficult to engage with and they grade the lesson very positively.

Quite clearly, the answers indicate that the students find MERIA scenarios *interesting*. A majority of students explicitly say that the lesson was more interesting than a traditional one. The number of students claiming this is

extremely high in Croatia (90%) and Slovenia (78%), while above half in Denmark (62%) and the Netherlands (58%). It is interesting to note that the lesson was graded less interesting and at the same time slightly less clear in the Netherlands and Denmark. Unfortunately, we could not inquire deeper into the relation of these two characteristics or control other factors related to the level of preparation or enthusiasm of the teacher.

As to *relevancy* (meaningfulness) of the problems, the situation is less convincing. In Croatia 70% of students answer that the problem is meaningful to them, but only 20% of the students in the Netherlands express in that way.

The number of students that say that they would like to have this type of lessons (Question 7) every week or once in two weeks in very high in Croatia (75%), Slovenia (70%) and Denmark (82%), while these two answers were chosen by 44% of the tested students in the Netherlands. The students mostly report that their level of engagement (Question 10) was the same or higher than compared to the regular lesson – less than 15% of students answered that they worked less.

As each of these characteristics was evaluated based on one question, we remind ourselves also that we have no way to know to what extent the significance of the results is reliable and did the students want to give expected answers.

It might be still of interest to analyse the words that the students used to describe the lesson. In Croatia, Slovenia and the Netherlands the most used words indicate that the students found the lessons *interesting* and *fun*, that they have recognized *working in groups* and *independently* (of the teacher). In Croatia and Slovenia, students noted using *computers*, while in Denmark students noticed the rich character of the problems ("compare ideas", "different angles", "open task"). In general, there is an impression that the students enjoyed the lesson as it brought a fresh new way of teaching with the roles of the students and the teacher changed. Looking into other (not so frequently) mentioned words might also give some confidence that the students recognized the benefits of inquiry based teaching as intended by the project team, e.g. "good content", "confusing", "challenging", "useful", "not in the book", "applicable", "independent thinking", "logical", "cooperation", "participation".

Dissemination

In the final part of this booklet we also provide all 12 newsletters (in English) that were shared with the teachers in all four countries (in local languages) during the project. Along with the main results, the project team has written several publications and disseminated them at various conferences on mathematics educations. The results of the analysis of the first round of semi-structural interviews in Croatia

B. Divjak, M. Bašić, E. Špalj, Ž. Milin Šipuš - MERIA – Are we ready?

were presented at the Colloquium for Geometry and Graphics in Fužine, Croatia, in September 2017.

Main theoretical ideas of the project

E. Špalj - MERIA – Mathematics Education – Relevant, Interesting and Applicable (Matematičko obrazovanje – značajno, zanimljivo i primjenjivo)

were presented at VIII International Conference 3C (CCC-Computer, Communication, Culture) in Novi Sad, Serbia in November 2017.

A research paper analysing the impact of the MERIA scenario Slide

R. Bos, M. Doorman, K. Cafuta, S. Praprotnik, S. Antoliš, M. Bašić - Supporting the reinvention of the slope of a curve in a point

presented in February 2019 at the Eleventh Congress of the European Society for Research in Mathematics Education in Utrecht, the Netherlands. The results were also preliminary presented at the RME Conference in Cayman Islands in September 2018. The subject was also discussed in the Netherlands at the Summerschool on mathematics education 2018 and 2019 (Utrecht University), MATRic conference on Calculus teaching 2019, U-Talentconferentie 2019 (UU), Research seminar 2018 (Freudenthal Institute, UU), OnderwijsMeetsOnderzoek (ResearchMeetsEducation) 2018 with various audiences of teachers, teacher trainers and mathematics education researchers.

Project MERIA was also presented by the project leader and other team members in the form of a lecture at:

1. County meeting of mathematics teachers in Split, Croatia organized by the Education and Teacher Training Agency in August 2017.
2. National meeting of mathematics teachers in Sv. Martin na Muri, Croatia organized by the Education and Teacher Training Agency in April 2018.
3. International Conference on Learning and Teaching Mathematics 2018 in Laško, Slovenia organized by the National Educational Institute Slovenia in June 2018.

The project has been presented as an example of good practice at a meeting for new users of the Erasmus+ programme by the Agency for Mobility and EU Programmes in Croatia in September 2017. Furthermore, the project has been promoted in the Croatian national radio and television on a few occasions in the news (reporting on the MERIA Final Conference in May 2019) and the following shows dedicated to educational and research themes: Znanstveni krugovi (March 2017), Prometej (May 2017), Znanost i društvo (September 2019).

MERIA activities and follow-up project

The evaluation of MERIA workshops and MERIA Final Conference clearly show that the participants have been highly satisfied. In particular, they appreciated being in an active role, obtaining materials they found useful and supported the main ideas of the project. These activities have been evaluated by the participants by filling out a simple questionnaire. The project team must be proud to see in the comments that some participants find the MERIA activities above the standard provided by professional development courses in their countries, unique in its concreteness, practicality and usefulness.

The same project team will continue to provide further professional development cycles and we hope that more scenarios will be produced in the near future. Along with the dissemination activities aimed towards teachers, the project team had organized six project meetings and three training weeks. The products of these meetings are not only the physical results of the project, but also a particularly fruitful collaboration of experts with different background in mathematics and mathematics education from all four countries. The training weeks served not only to preach about one's own point of view, but also to learn and compare different theoretical frameworks and understand affordances and constraints of our four national (and actually many more local) educational eco-systems. The activities of these three training weeks have been documented on the project webpage. Certainly, the best proof of the successful collaboration is the fact that all the results have been produced jointly by experts from all four countries, but also the fact that the same project team (with a few more schools) has applied for and received funding for a follow-up project called TIME.

The main idea of the project Teachers' inquiry on mathematics education (TIME) is to explore how a community of math teachers working together in one school can improve their practice through joint exploration, planning and interaction between themselves and with university professors. We will explore how such a community can reach a high level of independence and sustainability in creating innovative teaching and learning materials and upgrade their professional knowledge.

More concretely, the idea is to adapt the well-established format of Lesson study (LS) to conditions in the participating countries. LS is a Japanese model of shared inquiry, performed by small groups of teachers with a specific goal to improve their teaching, and thereby students' learning. In this project we propose to form LS teams of teachers in all partner countries. The teams will work on implementation of inquiry based mathematics teaching (IBMT). While there is an abundance of research showing effectiveness of LS at the primary and lower secondary level, especially in Japan and other East Asian countries, the model is new in Europe and especially at the upper secondary level.

The project TIME will draw on results and networks established by MERIA. MERIA showed that design of innovative teaching materials is an iterative process that requires expertise and time. Dissemination activities showed that teachers need such materials and moreover that they often do not believe they can produce them on their own. The first part of the project (TIMELESS) is dedicated to establishing LS among all partners. The second part (TIME^2) will shift the focus to teachers taking the role of designers, as it is the usual practice of independent Lesson study. In particular, teachers will learn how to use didactical phenomenology to develop new scenarios with rich and motivating mathematical contexts. During dissemination activities at the end of the project, experiences and materials will be shared to a wider public of teachers.

Purpose and goals of MERIA Project Impact Analysis

An overview of impact analysis objectives with methods of measuring results

Prepared by: Blaženka Divjak, Faculty of organization and informatics, University of Zagreb, Croatia

General idea of impact analysis

Generally, the purpose of an impact analysis is to evaluate the effects, both pro and con, of proposed ideas, solutions, material etc. It can be considered as a part of quality assurance (QA) system of an institution or project. Steps that can be followed to assess a project impact:

1. Set specific goals of the impact analysis,
2. Clearly defining the outcome or outcomes to be assessed,
3. Decide if it is feasible to observe an impact in the given timeframe (usually until the end of a project) for selected outcome(s) and selected environment or target groups,
4. Develop a baseline for evaluation (state of the art, status quo),
5. Build Project Impact Analysis Table (PIAT) with methods and indicators of impact measurement (connected with Logical Framework Matrix),
6. Follow PIAT during project lifetime, data collection and using data for QA;
7. Evaluate the project impact.

Overview of MERIA impact analysis

Following the steps from the previous section here is the overview of MERIA project impact analysis.

1. **Goals for MERIA impact analysis:**

A) To analyse the situation regarding *use of different teaching and learning methods, especially IBMT elements, in secondary schools mathematics classes* in participating countries and specially selected schools in order to:

a) Compare situation in different countries and schools to find out best practices.

b) Design interventions: workshops (based on the new "MERIA methodology") and teaching and learning materials and methods (TAL).

B) To evaluate the project impact on school level (selected schools) and teachers participating in different project's programmes and activities.

2. **Outcomes to be assessed**:

- Teaching and learning methods
- TAL material: Scenarios and Modules
- Workshops: quality of the workshop delivery and participants' approaches to TAL materials
- Success of collaboration in the consortium and beyond: assess the innovation process (for TAL) and dissemination of ideas

3. **Feasible to observe an impact of project MERIA**:
   - Change of *teaching and learning methods* will be evaluated only on the level of selected schools and only for classes taught by teachers that have been trained during project lifetime and that are using TAL prepared in the project or similar prepared by themselves
   - Scenarios and Modules will be evaluated by teachers, students and didacticians in the project team
   - Before each *workshop*, learning outcomes will be explicitly set, expectations of participants collected and after the workshop participants will fill out questionnaire about their satisfaction and meeting learning outcomes

4. **Baseline for evaluation**. The main tools for evaluation used in the MERIA project impact analysis are:
   a. Comparison of state of the art in partnering countries
   b. Analysis of semi-structural interviews
   c. Teacher evaluation based on a questionnaire
   d. Student evaluation based on a questionnaire

5. **Project Impact Analysis Table** (PIAT) with methods and indicators of impact measurement is given in Table 1.

   The following *specific objectives of the project MERIA* are referred to in PIAT:
   1. To promote mathematics education that is student-centred, inquiry-based, guided by discovery of mathematics, relevant to everyday life that promotes deeper learning approach, power of mathematical modelling and abstract thinking;
   2. To provide systematic and overarching support to the curriculum, by creating a repository of mathematical showcase scenarios and modules for secondary schools;
   3. To upgrade teaching skills of in-service teachers so that they have necessary competences for dealing with diversified groups of learners and to tackle problem solving and mathematical modelling by using learner-centred approaches and methods;
   4. To strengthen cooperation and flow of ideas between educational and research institutions dealing with mathematics education at different levels as well as transfer of innovative practices at European level.

6. **Work group** (WG) will be in charge of **following PIAT during project** lifetime, data collection and using data for QA. WG will report to coordinator and the Steering Committee on regular bases (intervals will be defined at the kick-of meeting, proposed every 6 months)

7. WG will prepare **an evaluation** of the project impact near the end of the project.

Specific objectives	Outcome	Indicator of success	(Method of) Measuring indicators	Responsible for collecting data, analysis	Time frame	Risks
1. To promote student-centred, inquiry-based mathematical education	PRACTICAL GUIDE FOR IBMT (Followed by upgraded classroom environment)	-Change of math TAL -Attitudes of teachers and students -Satisfaction of students	-Document analysis (comparison) -Semi-structural interviews with teachers	University of Copenhagen leading WP2 (methodology)	M6-M12 M13-M27 M33-M36	If "good" schools and teachers are chosen familiar with promoted approach
2. To provide a repository of mathematical showcase scenarios and modules for secondary schools	SCENARIOS & MODULES	-Compliance with proposed pedagogy -Usability for TAL -Effectiveness -Attractiveness	-Expert evaluation (pedagogical soundness) -Teacher evaluation (usability in practice) -Student satisfaction	University of Ljubljana leading WP3 (scenarios and modules) Teacher and student evaluation: *selected schools*	M13-M27 M13-M27 M32	Lack of interest for new materials. Misunderstand-ing between the groups designing pedagogy and materials. Misconceptions what type of TAL materials students find attractive.
3. To upgrade teaching skills of in-service teachers	WORKSHOP GUIDE & WORKSHOPS	-Meeting learning outcomes -Satisfaction of participants	-Expert evaluation of the workshop guide - Questionnaire after the workshop	Utrecht University leading WP4 (workshops)	M29 M32 M29 M32	Wrong assumptions on teachers' skills and needs due to flaws in needs analysis
4. To strengthen cooperation between educational and research institutions dealing with mathematics education	PROJECT MEETINGS & TRAINING ACTIVITIES	-Meeting criteria set for methods and materials -Scope and scale of impact of project results	-Reports of leaders of WPs -Publications	Faculty of Science, University of Zagreb leading WP1(impact analysis) and WP5 (quality assurance)	M4 M11 M16 M24 M28 M33	Different styles of work Lack of partners' personal commitment Misconceptions about mutual expectations Difficulties in communication

Table 1. Project Impact Analysis Table (PIAT)

State of the art in mathematics education in partnering countries – questionnaire for researchers

Prepared by: Matija Bašić, Faculty of Science, University of Zagreb, Croatia

Purpose and format.

In this analysis we will produce an overview (state of the art) and comparison of educational systems (concerning mathematics) in partnering countries with the emphasis on aspects relevant to the project. Chapter is organized into five domains and each domain is described in two-three pages. Domains are chosen according to reference [1] so that we can cite relevant existing reports for Slovenia, Netherlands and Denmark, as well as recommendations of the European Commission in the field of mathematics educations. Also, we follow main domains for the semi-structured interview of teachers. Hence this chapter serves as an introduction and a reference point for the impact analysis, design of materials and workshops.

Domains.

We will organize the collection of data according to the following questions in the five domains.

1. Teachers' initial education and professional development
    a. Do all mathematics teachers working in high schools have the same degree? From which Faculties? **(Ch. 6 in [1])**
    b. What are the possibilities for professional development of teachers? Which institutions are involved in organization of workshops for teachers? **(Ch. 6 in [1])**
    c. Is the collaboration between high schools and higher education institutions intensive? What are examples?
2. Teaching approaches, methods and materials
    a. What are preferred teaching methods? Are there national rules for writing lesson plans and preparing the course? **(typical examples as discussed at the Kick-off meeting; Ch. 2 in [1], [3])**
    b. What are the topics expected to be covered by the national mathematics curriculum in schools where students (on average) have 3 hours of mathematics per week? **(Annex 1 and Chapter 1 in [1]; also [4] for outline in some countries)**
    c. Is there a variety of text books supporting different approaches of teaching mathematics? Are there other approved/verified materials?
3. Inquiry-based approach
    a. Is there a tradition, research and literature in the national language on using IBME?
    b. Are there surveys on attitudes towards, usage and impact of IBME? What do these studies focus on and what are the results? **([5], [6])**

4. Assessment
    a. What is the form of the graduation exam? **(Ch. 1 in [1])**
    b. Are there formal rules/guidelines/recommendations for assessment? **(Ch. 3 in [1])**
    c. Is there any research on assessment, in particular of higher cognitive skills?
5. Students' achievement
    a. What are the results of PISA (and TIMSS advanced)? **([2])**
    b. Are there other surveys that have comparable results?
    c. Are there initiatives that address low achievement in mathematics or increase students' motivation? **(Ch. 4 and 5 in [1], [3])**

Literature

[1] Mathematics Education in Europe: Common Challenges and National Policies, published by EACEA P9 Eurydice, European Commision, October 2011.
Remark: Study includes EU members in 2010/11, which excludes Croatia!
[2] PISA: Results in Focus, OECD 2016.
[3] Ten Questions for Mathematics Teachers… and How PISA Can Help Answer Them, OECD 2016.
[4] Description of the Advanced Mathematics Programs and Curriculum, TIMSS ADVANCED 2015
[5] The Significance of TIMSS and TIMSS Advanced - Mathematics Education in Norway, Slovenia and Sweden, Goenmo et al., Akademika Publishing, 2013.
[6] Mascil – Report on the large-scale survey about inquiry based learning and teaching in the European partner countries, 2014.

Working plan

Step	Description	Deadline
1	Comment on the questions under "Domains" using GoogleDocs	February 2017
2	Members of the Work Group gather additional documents and answer questions under "Domains" for their country	March 2017
3	First draft of the report	May 2017
4	Comments on the first draft by Work Group	June 2017
5	Final version or the report	July 2017

State of the art in mathematics education in partnering countries - overview and comparison based on existing data

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Introduction

Project MERIA aims to address students' low proficiency and motivation in mathematics by promoting student-centred approaches in mathematics education with positive attitudes towards mathematics and its applications. Today, national curricula worldwide more strongly than before recommend implementation of innovative teaching methods in everyday teaching, emphasizing more active role of a student. We hypothesize that inquiry based mathematics teaching (IBMT), with little guidance of the teacher and students' work similar to those of mathematicians, enhances student's motivation, brings experience of mathematics as relevant, interesting and applicable for their lives, and thus contributes to our goal. In this teaching approach, project team also recognizes the importance of teachers' role (Sanders, W.L. et al., 1997) and strives to provide sustainable support in the form of innovative teaching and learning materials and through teachers' professional development.

Regardless of these visions, many questions still arise connected to implementation of IBMT in classroom practice – e.g. whether such teaching approach is beneficial for all students, if students' pre-knowledge is sufficient, how to deal with students' heterogeneity, how IBMT is aligned with assessment, whether it is worth of teachers' efforts, what kind of support teachers need, who offers this support, etc. These and other various aspects of IBMT have been studied in numerous research projects worldwide at all levels of education. In fact, traditional teaching approaches have not undergone similar investigation. Positive effects of IBMT are primarily described by better students' motivation, deeper understanding of mathematical topics, and by development of beliefs about the relevance of mathematics for life and society (Bruder & Prescott, 2013), or by their increased capacity to think critically, especially those students who have not been previously encouraged that way (Hattie, 2009). On the other hand, institutional restrictions for implementation of IBMT, on the level of society, school, pedagogy and discipline (Dorier & Garcia, 2013) are recognized in e.g. the lack of school hours (it is seen as time-consuming), teachers' initial training (in

which IBMT does not dominate, nor deeper understanding of mathematics is shared by preservice teachers), in in-service training (which is loosely organized and does not focus around experiencing IBMT), official orientation in pedagogy, national assessment, etc.

However, promotion of inquiry based mathematics teaching need not be opposed to other teaching approaches, like traditional teaching (transmission of knowledge). It is possible to combine different approaches and to include IBMT only once in a while (Dorier & Garcia, 2013). Adoption of an intermediate position between teacher's transmission of knowledge and students' inquiry could be optimal (Godino et al., 2015). "The best teaching will often integrate ideas from constructivist and behaviourist principles" (Steele, 2005).

In this chapter we present a brief overview of educational systems in partnering countries, by emphasizing aspects relevant to the project. Institutional aspects might hinder implementation of IBMT, or can favour and provide good conditions for its implementation (Dorier & Garcia, 2013). This chapter therefore serves both as an introduction and as a reference point for the project's impact analysis, and for design of innovative materials and workshops.

MERIA project is oriented on the practical side of promoting IBMT and providing support to secondary school math teachers. Development of this support is based on a needs analysis presented in this booklet. This chapter has been written based on the method of document analysis and presents main aspects taken into account when considering implementation of IBMT. Data presented here is collected from recent major reports on mathematics education within four partnering countries and the wider European educational space represented by OECD countries. As is often the case in document analysis, we rely on the description and interpretation of data rather than having the raw data as a basis for analysis (Bowen, 2009, Labuschagne, 2003).

We find the following aspects of educational systems relevant for our study and we organize the chapter accordingly:

Students' achievement in mathematics at large scale assessment 22
Types of secondary schools and programmes 26
Teaching approaches, methods and materials 30
The role of teachers in MERIA 41
Teachers' initial education 43
Teachers' professional development 46
Assessment of mathematics achievements of high school students. 50

Domains are chosen and the text is mostly streamlined by following the report *Mathematics Education in Europe: Common Challenges and National Policies*, published by EACEA P9 Eurydice (European Commission) in October 2011 (Eurydice Network, 2011). In addition, the organization into these main domains is similar to those for semi-structured interviews of teachers that is used in the main part of the project's impact analysis. Thus, the organization enables subsequent comparison and matching of regulations, recommendations and our expectations to the existing teachers' actual practice in the schools associated to the project.

Students' achievement in mathematics at large scale assessment

Project MERIA aims to address underachievement of students in mathematics and thus contribute to overall aim of Strategic framework for European cooperation in education and training (ET 2020) to reduce the rate of low achievers from 22% to 15%. The aim of the project is not restricted to design of actions for students in the partnering countries only, but also to provide new pathways and frameworks that could improve mathematics education wherever student-centred methods are not developed and implemented. Nonetheless, we focus on the four partnering countries since our materials and other results are planned to be tested there only.

National curricula worldwide nowadays are less content- and more competence-oriented. International PISA assessment contributed to this movement and thus created "favourable conditions for the adoption" which "can be interpreted as opportunities for spreading" of IBME (Dorier & Garcia, 2013, p. 842). We start with the overview of results of PISA mathematics assessment (OECD, 2018). PISA assessment is conducted with 15-years old students, giving thus information on students prior their educational level which is target level of the project MERIA (lower and upper secondary school).

Country	Mean score	Mean score M / F
The Netherlands	512	511 / 513
Denmark	511	506 / 516
Slovenia	510	508 / 512
Croatia	464	458 / 471
OECD	490	486 / 494

Table 1. Result in mathematical literacy PISA 2015

The Netherlands scored position 11 in PISA 2015, thus being in Europe's sub-top. Regardless of this rank, its score points have been decreasing since 2003, as well as the OECD average score has. In addition, the number of mathematics lessons per week is among the lowest when compared to other partnering countries. This suggests that mathematics education in the Netherlands still is quite effective. The recent decline in skill score in the Netherlands might also be due to an increasing

emphasis on basic skills (at the cost of higher order reasoning skills) (CITO, p.13). The difference between boys' and girls' performance is not observed.

Country	Low performers (M / F)	Top performers (M / F)	Average trend
The Netherlands	16.7 (17.2 / 16.2)	15.5 (17.0 / 14.1)	-6
Denmark	13.6 (12.9 / 14.2)	11.7 (13.7 / 9.6)	-2
Slovenia	16.1 (16.1 / 16.1)	13.5 (14.9 / 12.0)	2
Croatia	32.0 (30.0 / 33.9)	5.6 (7.1 / 4.1)	0
OECD	22.9 (22.6 / 23.2)	10.8 (12.6 / 9.0)	-2

Table 2. Percentage of low and top performers

It is relevant to notice that PISA results suggest that student interest in science and mathematics in the Netherlands is not high. PISA 2012 shows that students' motivation in the highest in Denmark, then followed by the Netherlands and Croatia, and finally the lowest in Slovenia.

Denmark's and Slovenia's PISA 2015 average scores in mathematical literacy just follow the Netherland's, and therefore are still significantly higher than the OECD average. In Denmark, a basic level of mathematical literacy (level 2) in PISA 2015 achieves 86% of the students, whereas in Slovenia 84%. The highest levels (level 5 and 6) In Denmark are achieved by 12% of the students and in Slovenia by 13%. On average in OECD countries boys are more successful in the mathematical literacy than girls. Achievements of the Danish and Slovenian 15-year-old students show no statistically significant differences between the sexes.

With the score of 464 points, Croatia is significantly below OECD average and the results of other countries. Results of PISA 2012 show that Croatia has a high share of 29.9% of low-achievers in mathematics (OECD average 23.1%), low share of 7% of top-performing students (OECD average 12.6%) and the annual change in score points of 0.6. PISA 2015 shows stable trends in performance results with the annual change in score points of 0, although the share of low-achievers increased to 32.0% and top performers decreased to 5.6%. The difference between boys' and girls' performance is observed throughout these years, in favour of boys.

Country	Average time per week in math lesson	Average time per week in math after school	Average time per week in studying after school
The Netherlands	2.6	2.6	14.2
Denmark	4.2	3.6	18.7
Slovenia	2.7	4.1	17.3
Croatia	2.5	4.0	19.8
OECD	3.6	3.8	17.1

Table 3. Time students spend learning math (in hours per week) according to PISA

A natural question for the MERIA project team is how to address these issues and promote students' interest and positive attitude in mathematics for various groups of students?

"Countries often use analyses of PISA and TIMSS data to assess mathematics performance and identify the reasons for students' low achievement. In some cases, these analyses are supplemented by reports based on results from national standardized tests. In both cases, conclusions point to the fact that underachievement in mathematics occurs for a number of reasons linked to home background and school-related factors that often reinforce each other." (Eurydice Network, 2011, p. 81-82)

Addressing the low achievement in mathematics is an important issue in all partnering countries. For example, in Slovenia some national researches report on the correlation of results from national examinations with the socio-cultural environment of high school students, classes and homework (Žakelj, 2013). Similarly, Eurydice Network, 2011, report recommends "responding to the diverse needs of learners" and "focusing on individual weaknesses", while (OECD, 2016) emphasizes that "for disadvantaged hours of work does not equate results as for advantaged" and that "disadvantaged are less frequently exposed to both applied and pure mathematics".

A first step in dealing with low achievement might be improving engagement and motivation of learners. There are various suggestions coming from the Slovenian educational system. Apart from extra-curricular activities where students are encouraged to participate in various projects, there are specific teaching methods for increasing engagement. They mainly focus on the use of innovative teaching methods including the use of ICT. Furthermore, more attention is given to organization of campaigns for promoting mathematics in a popular and humorous way (like the TV-show Bite into Science, http://4d.rtvslo.si/oddaja/ugriznimo_znanost/65345770. Finally, scholarships for the best students provide additional motivation. It is also noticed that some students enjoy competitions, therefore a question arises if it is so also with students with lower achievement in mathematics and whether this way also provides motivation.

"In addition, national studies provide data on problematic subject content and mathematical skills. For instance, algebra, mathematical communication, and problem solving in context have been identified as common problematic areas for students. Not surprisingly, the same content areas presented difficulties for teachers." (Eurydice Network, 2011, p.83)

Project MERIA will develop a more standard approach in the sense that it provides (new) classroom activities, addresses majority of students and covers topics from the national curricula, but at the same time it will shift focus from traditional teacher-directed methods to more student-centred methods, by changing the role of teachers from lecturers to activators (Hattie, 2009).

"Teaching methods should address the perceptions that mathematics is difficult, abstract or uninteresting and not relevant to real-life. One way of doing so is to organize lessons around 'big ideas' and interdisciplinary themes that help establish connections with everyday life and other subjects. This approach is at the core of the well-established 'Realistic Mathematics Education' programme in the Netherlands (Heuvel-Panhuizen, 2000) ." Another approach, that of Theory of didactic situations which was developed in France and practiced in many countries, emphasizes the idea of student's interaction with an appropriate milieu within which the problem is posed, and which enables the student to progressively build mathematical knowledge at stake (Brousseau, 1997). The teacher's role is therefore to design such a milieu. Both approaches are implemented in MERIA activities.

Conclusion

International PISA assessment contributed to the worldwide movement of promotion of less-content and more competence-oriented curricula which "can be interpreted as opportunities for spreading" of IBMT. PISA assesses 15-years old students, giving thus information on students prior their educational level which is the target level of the project MERIA (lower and upper secondary school). Students in partnering countries show various achievements in PISA testing. While scores in mathematical literacy for the Netherlands, Denmark and Slovenia are very similar, and in the Europe's sub-top (scores between 510 and 512), Croatian students perform significantly lower (score 464, while the OECD score is 490). These results are due to many factors, with none of factors exemplified here (classroom environment results, like the average time per week spent learning in regular math lesson, in math after school or studying after school) being significantly influential.

Low achievement in mathematics education may be addressed by innovative teaching methods, popularization activities and other activities (e.g. participation in projects and different forms of competitions) that increase students' motivation. Surveys show that innovative teaching methods, e.g. inquiry-based mathematics education (IBME) increases students' interest for mathematics. Positive effects of IBMT are primarily described by better students' motivation, deeper understanding of mathematical topics, and by development of beliefs

about the relevance of mathematics for life and society. We further hypothesize that IBME also enhances deeper understanding and retention of learned knowledge. Special attention in MERIA is given to ways to support development of research-like activities in diversified student body, and organization of classroom activities that provide cognitive and emotional activation to more students.

MERIA team chose to investigate the difficulties that teachers face and challenges that are common to all of them. In associated schools, teachers are asked to participate in semi-structured interviews to express the problems they are confronted with in their daily practice, and their opinions on the changes needed to support learners in their classrooms.

Types of secondary schools and programmes

MERIA project focuses on secondary mathematics education. To perform a needs analysis and to evaluate new materials, MERIA will collaborate with the associated schools in each partnering country. The profile of these schools is chosen by the number of hours that students learn mathematics.

In **Denmark** primary school is 9 years with an optional 10th year. There is no separate lower secondary division. Upper secondary school is offered in several versions: STX (general education), HTX (technical education), HHX (mercantile education). These are 3-year educations. HF is a 2-year general education (aiming at students with some working life experience), and EUX is a 4-year education combining high school with a vocational education, say within a craft. The Danish MERIA project only involves STX, HHX and HTX, so we shall concentrate the description to these.

In STX and HHX there are 3 levels A, B, and C of mathematics (and other disciplines as well). A law revision for upper secondary education was passed by the parliament in 2016. It includes rules for combining levels. However, C-level is only permitted if the student has a heavy load of language subjects at A-level. HTX has no C-level in mathematics. There is a flat entrance to high school. During the first ¼ of a school year the students select their levels. In addition to teaching hours there is so-called 'thorough study hours', that is assigned to independent student work.

There are 40 weeks per year of teaching in primary school equivalent to 200 teaching days. In high school there is 35 weeks per year of teaching.

Education level		Duration	Teaching hrs per year	Thorough study hrs per year
Primary education[1]		9 years	150	?[2]
Secondary education	STX A	3 years	375	1330
	STX B	2 years	250	85
	STX C	1 year	125	40
	HTX A	3 years	410	130
	HTX B	2 years	285	85
	HHX A	3 years	375	130
	HHX B	2 years	250	85
	HHX C	1 year	125	40

Table 4. Primary and secondary level of education in Denmark

For general information on the **Dutch** educational system see e.g. (Nuffic, 2015) from which the table below is extracted. MERIA activities will focus on two special types of secondary education - vwo (University Preparatory Education) and havo (Senior General Secondary Education) education. Most children start preprimary education which lasts for two years at the age of 4. Preprimary and primary education consists of eight grades. Secondary education in The Netherlands begins at the age of 12.

Education level		Duration
Primary education		7-8 years
Secondary education	Preparatory vocational secondary education (vbmo)	4 years
	Senior general secondary education (havo)	5 years
	University preparatory secondary education (vwo)	6 years

Table 5. Primary and secondary level of education in the Netherlands

[1] 150 is the minimum requirement. Each school is allocated a (small) pool of teaching hours to distribute in accordance with the school's intended profile.
[2] This varies a lot from school to school and even from teacher to teacher due to their employment agreement.

Upper secondary education in **Slovenia** is very diverse with different programmes and different numbers of math lessons per year as shown in Table 6. The number of math lessons in individual year classes is not prescribed with the curriculum on the state level, but the decision regarding the distribution of them is solely left to the individual school. There are different topics covered in different programmes.

Education level		Duration	Total math lessons for the programme duration
Primary education			
Secondary education	Upper secondary general education (gimnazija)	4	560
	Upper secondary technical education	4	383-408
	Upper secondary vocational education	3	206-242
	Vocational-technical education	2	213
	Short upper vocational secondary education	2	157

Table 6. Primary and secondary level of education in Slovenia

In **Croatia** there are three types of secondary schools: gymnasiums, vocational and art schools. Education in gymnasiums and art schools lasts for four years while vocational education can take either three or four years. There is a large variety of programmes. They differ in number of hours of mathematics per week, from none in some grades in vocational and art schools to seven hours in one of the programmes offered in mathematical gymnasium. In gymnasiums that provide general education there are four hours of mathematics in the first two years and three hours in the last two years; 481 hours in total.

Education level			Duration	Number of math lessons per week[3]
Primary education			8 years	4-5
Secondary education	Gymnasiums	Science	4 years	5-7
		General		3-5
		Linguistic		3-4
		Classical		4
	Vocational schools – 4 years programmes	Technical	4 years	3-5
		Commercial		2-4
		Touristic		3-4
	Vocational schools – 3 years programmes		3 years	1-2
	Art schools		4 years	1-2

Table 7. Primary and secondary level of education in Croatia

Conclusion

Scenarios and modules for teaching, developed in the project MERIA, are tested in associated schools. These schools are chosen among similar programmes in each country, having 3-4 hours of mathematics per week. In Denmark, the associated schools will involve the Upper Secondary schools (STX, HF, HHX and HTX). In the Netherlands these are the vwo (University Preparatory Education) and havo (Senior General Secondary Education) schools. In Slovenia MERIA's focus will be on the Upper secondary general education (gymnasium) and Upper secondary technical education. In Croatia the associated schools will be selected among general gymnasiums, engineering/technical vocational schools and commercial high schools. Number of math lessons in different schools varies according to the programmes and different school years. Mathematics is not a compulsory subject in vocational school in all school years. Teachers in different school programmes with mostly heterogeneous student body, meaning with students with different pre-knowledge, are in special need for appropriate support in implementation of IBMT.

[3] Number of math lessons varies according to a programme and school year. Math is not a compulsory subject in vocational school in all school years.

Teaching approaches, methods and materials

Across the world mathematics education is based on different learning theories and teaching approaches. These are reflected in the teaching methods and materials that teachers use. The national standards for teaching are prescribed by core documents that present the curricular system, but the teaching approaches and the level of details that are prescribed or expected vary from country to country. Insight into teaching practices is provided mostly by questionnaires in international studies such as PISA, TIMSS and TALIS. This evidence points out that traditional transmissive practices predominate in everyday classes in many countries. Consequently, this dominant teaching practices might act as a constraint for the implementation of IBMT. Furthermore, teaching materials, mostly textbooks are widely recognized as the important material for design of a lesson. Shaping them to follow ideas of IBMT is necessary in providing support for implementation.

"The improvement of mathematics education depends largely on new textbooks. They play a determining role in mathematics education. Actually, textbooks are the most important tools guiding the teachers' teaching. This is true of both the content and the teaching methods, although with regard to the latter, the guidance provided is not sufficient to reach all teachers." (Heuvel-Panhuizen, 2000)

Project MERIA focuses on providing support to mathematics teachers by designing new teaching materials that supports both the national curricula as well as and extracurricular mathematics topics. Furthermore, it will provide teacher training in the student-centred inquiry-based approach following two well-established theories in mathematics education, RME and TDS. In order to design materials that teachers will find useful we have asked ourselves: What kind of textbooks and other resources of problems do teachers already use? Which mathematics topics are covered in all partnering countries and which aspects are missing? Do teachers base their teaching methods in particular theoretical approaches? Are there strict rules given by the national documents that teachers must follow?

To answer these questions, we provide a preview of topics covered in the national curricula accompanied by specific details concerning the use of different methods and materials in each partnering country.

Denmark

There are several textbooks to support different approaches, but the inquiry-based approach is not represented in high school. Teachers share teaching materials through various platforms or use websites with materials developed through projects. The Ministry of Education (www.EMU.dk) facilitates the teachers, the Association of Math teachers informs teachers via newsletters, the

website www.mat.dk and the magazine *LMFK-Bladet*. The teachers share their materials on local intranets and in addition to this, there are also private initiatives (e.g. steen-toft.dk, uvmat.dk or www.matematiksider.dk).
In general, there are no required teaching methods, but the curriculum sets certain demands and recommendations, e.g. experiments, simulations and constructions on A- and B-level also using computer algebra systems (CAS). The study plan stipulates intended competencies, called subject aims and curriculum topics. These are divided into core curriculum and supplementary curriculum. Details of topics within curriculum of the various levels are available at https://uvm.dk/gymnasiale-uddannelser/fag-og-laereplaner/laereplaner-2017

STX A	STX B	STX C
Rough calculations, the hierarchy of basic arithmetical operations, symbol manipulation, even and inverse proportionality, the extended power concept, equation solution with algebraic and graphical methods, and numerical methods using mathematical software, approximate and exact value as well as absolute value	Same as STX A	Rough calculations, hierarchy of basic arithmetical operations, simple symbol manipulation, even and inverse proportionality, equation solution with algebraic and graphical methods
percentage and interest rate, absolute and relative change, interest rate formula	Same as STX A	Same as STX B
statistical methods for handling discrete and grouped data, graphical presentation of statistical material, sample and empirical statistical descriptors, as well as the use of linear, exponential, power and polynomial regression including uncertainty considerations and residual plot	simple statistical methods for handling discrete and grouped data, graphical presentation of statistical material, sample and empirical statistical descriptors, as well as the use of linear, exponential,	simple statistical methods for handling discrete and grouped data, graphical presentation of statistical material, sample and empirical statistical descriptors, as well as the use of linear, exponential and power regression, including residual plot

	power and polynomial regression including uncertainty and residual plot	
combinatorics, basic probability, probability field and stochastic variable, binomial distribution and normal distribution, confidence intervals, hypothesis test in binomial distribution	combinatorics, basic probability, probability field and stochastic variable, binomial distribution including approximation with normal distribution, confidence intervals, hypothesis test in binomial distribution	combinatorics, basic probability, symmetric probability field
function concept, composite function, piecewise defined function, inverse function, characteristic properties of the following elementary functions and their graphs: linear functions, polynomials, exponential, power and logarithmic functions, and trigonometric functions	Same as STX A, but only graphical handling by mathematical software of simple trigonometric functions and their properties.	function concept, characteristic properties of linear, exponential and power functions and their graphs
definition and interpretation of differential quotient, including growth rate, derivative function of elementary functions, and rules for differentiation of sum, difference and product of functions and differentiation of composite function	Same as STX A	Graphical handling of second-degree polynomials and logarithmic functions and their properties in a mathematical computer programme.

monotony, extremals and optimization, and the relationship between these concepts and the concept of differential quotient	Same as STX A	Graphical determination of tangent, monotony intervals and extremals for functions defined on bounded intervals
vectors in two dimensions given by coordinates, including scalar product, determinant, projection, angles, area, line, circle, intersections and distance, and uses of vector-based coordinate geometry for the formulation and resolution of planar geometry problems, including trigonometric problems	Same as STX A	Same as STX B
vector functions, graphs of paths of motion, including tangent determination, and uses of vector functions	None	None
Functions of two variable, partial derivative and graphs, including level curves	None	None
Antiderivatives of elementary functions, indefinite and definite integrals, the relationship between area and antiderivative, rules for linearity of integration, and integration by substitution, applications of integrals	None	None
First order linear and separable differential equations, including the logistic differential equation, qualitative analysis of differential equations, and setting up simple differential equations	None	None
Characteristic principles of mathematical models,	Characteristic principles of	Same as STX B

mathematical modeling, including the use of some of the above-mentioned function types and combinations thereof, as well as modeling using derivative function.	mathematical models, mathematical modeling, including the use of some of the above-mentioned function types and combinations thereof	

Table 8. Topics of core curriculum in Denmark by the national mathematics curriculum from 2017

Subject aims are described by what the students must be able to, for example:

STX A	STX B	STX C
operate with numbers and representations of numbers as well as critically evaluate the results of such operations	Same as STX A	Same as STX B
handle formulas, be able to develop and explain symbolic descriptions of variable relations and be able to use symbol-based language to solve mathematical issues	Same as STX A	handle simple formulas, be able to develop and explain symbolic descriptions of simple variable relation, and be able to use symbol-based language to solve simple mathematical issues
translate between the four representation forms table, graph, formula and linguistic description	Same as STX A	Same as STX B

Table 9. Excerpt of subject aims in Denmark by the national mathematics curriculum from 2017

The Netherlands

There are no officially preferred teaching methods, nor national rules for writing lesson plans and course preparation. School inspectors, who visit schools once in a while, may have preferences and seem to have stimulated group work over the last years, at the cost of whole-class teaching, but this is not more than an impression and no official policy.

The full curricula (in Dutch) can be found at College voor Toetsen en Examens (2018) or (in English) TIMSS Encyclopedia (2015). For the primary school, mathematics curriculum comprises 11 core objectives, and for the first two years of secondary school (till grade 8) 9 core objectives ("TIMSS Encyclopedia," 2015). Core objectives for first two years of secondary school are:

- Use appropriate mathematical language to organize mathematical thinking, explain things to others, and understand explanations in the context of mathematics
- Recognize and use mathematics to solve problems in practical situations, both individually and in collaboration with others
- Establish a mathematical argument and distinguish it from opinion, learning to give and receive mathematical criticism and to respect other ways of thinking
- Recognize the structure and coherence of the systems of positive and negative numbers, decimal numbers, fractions, percentages, and proportions, and learn to work with these systems meaningfully in practical situations
- Make exact calculations, provide estimates, and demonstrate an understanding of accuracy, order of magnitude, and margin of error appropriate to a given situation
- Make measurements, recognize the structure and coherence of the metric system, and calculate with measurements in common applications
- Use informal notations, schematic representations, tables, diagrams, and formulas to understand connections between quantities and variables
- Work with two- and three-dimensional shapes and solids, make and interpret representations of these objects, and calculate and reason using their properties
- Learn to describe, order, and visualize data systematically, and to judge data, representations, and conclusions critically

Textbook series are produced by commercial publishing companies and need no approval by national authorities. In practice, two series, published by the same publishing company, dominate the market with an estimated market share of about 90%. The two series are slightly different with respect to didactical approaches, the biggest one favouring a traditional approach and the smaller one offering somewhat more room for exploration and discovery. In addition, other commercial and non-commercial textbook series are available. In these categories a teacher can find resources in which digital tools play a more important role or in which a more explicit orientation towards reinvention and inquiry is chosen.
In the Netherlands, teachers are fairly free in their teaching, and the schools decide by themselves which textbook series they use. Textbooks are the most

important tools guiding the teachers' teaching. Teachers also use the ideas for teaching activities from the developmental research at the Freudenthal Institute and the SLO, the Dutch Institute for Curriculum Development (Heuvel-Panhuizen, 2000). There is a great heritage of Freudenthal's ideas and RME, however "many studies revealed indications that the implementation of RME in classroom practice is still not optimal" (Heuvel-Panhuizen, 2000), although its use is increasing. Recent evidence also showed that students' proficiency in basic mathematical skills fell short, and therefore subsequent discussion (Gravemeijer, Bruin-Muurling, Kraemer, & van Stiphout, 2016) encourages for efforts to promote more advanced conceptual mathematical understandings as key curriculum goals and integral parts of instructional sequences.

Slovenia

Curriculum in Slovenia is written in terms of expected outcomes that a student achieves given in the form of a list of content and procedural knowledge. Procedural knowledge outcomes include general skills and processes linked to mathematical knowledge and transferable to other areas as well.

"Suggestions and guidelines for connecting the topics with material from other academic areas are also included, as well as instructions on how the topics could be presented and taught in these contexts. There are some topics classified as optional or as left to the teacher's discretion based on the teacher's expectations for students' achievement." (TIMSS Advanced, 2015).
When preparing for class teacher is autonomous in choosing learning approach and learning methods. Rules specify that every teacher should document her work with an annual work plan and lesson plans. There are no rules how to do it but usually lesson plans include information on learning goals for the lesson, learning methods, learning approach, student activities and didactical material.

PISA survey indicates that in Slovenia students report on teacher-directed instruction in 66,77%, and on student-oriented instruction in 17,66% of cases. These results indicate that teachers could give learners a more active role in the lesson and include activities such as inquiry-based learning activities, student working on project in small groups and solving problems.

Textbooks that are available for mathematics education have been approved by Council of experts of the Republic of Slovenia for General education and they are in accordance with math curriculum for different programmes. Teachers are autonomous in choosing textbook for their students among valid/approved set of textbooks. Textbooks are written in a way that they meet students' needs and abilities and a student can learn from it in an independent way. E-textbooks are also available on https://eucbeniki.sio.si/ covering the first three years in Upper

secondary education. There are also other learning materials available for teaching and learning like workbooks, collections of exercises and other optional learning materials, but they are not approved by Council of experts of the Republic of Slovenia for General education since 2006.

Content Area	Upper secondary technical education (383-408 hours in 4 years)	Upper secondary general education (560 hours in 4 years)
Sets and Logic	Basics of logic; sets	Basics of logic; sets
Numbers	Number sets with whole, rational, real (complex numbers are optional topics), powers and roots	Number sets with whole, rational, real, and complex numbers (mathematical induction and the trigonometric/polar form of complex numbers are optional)
Geometry	Lines, angles, circles and triangles; sines and cosines; the areas of 2-D geometric shapes and the volumes of 3-D shapes and sections	Lines, angles, circles and triangles in a plane and in space; sines and cosines; areas of 2-D geometric shapes and the volumes of 3-D shapes and sections; Cartesian coordinate systems; vectors in a plane and in space, scalar product (vector product is optional)
Functions, equations	Cartesian coordinate systems; linear functions; systems of linear equations; quadratic, polynomial, exponential, rational, logarithmic and trigonometric functions; equations and inequalities and their methods of solution	Equations and inequalities and their methods of solution (parametric equations are optional); powers and roots; limits, continuity, inverse and composite functions; linear functions; solving systems of linear equations; quadratic, exponential, rational, logarithmic and trigonometric functions; conic sections
Sequences and Series	Arithmetic sequences, geometric sequences	Arithmetic sequences, geometric sequences and series
Calculus	Limits, continuity, derivation	Differential calculations; integrals; applications of integrals
Probability and Statistics	Probability and statistics	Combinatorics Probability and statistics

Table 10. Topics covered in Slovenia in schools within 4 years programmes

Croatia

There are few textbooks sets that are offered on the national level. Other approved material mostly comprises e-material offered by the Croatian Academic and Research Network – *CARNet* (https://www.carnet.hr/en). The following table shows the topics covered in most textbooks.

Content Area	Topics
Numbers	Natural numbers, integers, rational, real and complex numbers (algebraic and trigonometric/polar form), mathematical induction
Algebra	Algebraic expressions, algebraic fractions, equations and inequalities, powers and roots, binomial theorem
Geometry	Plane geometry, space geometry, plane trigonometry, figures and solids, Cartesian coordinate systems, analytic geometry, vectors in plane and in space, scalar product, conic sections
Functions	Linear, quadratic, exponential, rational, logarithmic and trigonometric functions, definition, composite and inverse functions and properties, graphs
Sequences and Series	Arithmetic, geometric, limits
Calculus	Derivatives and applications, integrals and applications of integrals

Table 11. Mathematical topics in Croatian textbooks

Although teaching methods are completely teacher-dependent, one can say that preferred teaching methods are teacher directed methods, comprising direct instruction and a teacher-student dialogue. There are no national rules for writing lesson plans and preparing the course, although lesson plans developed by pre-service teacher student follow a certain pattern. In period of two years after completing their studies, novice teachers are obliged to pass the teacher state exam. Lesson plans developed for this exam usually follow similar rules.

In the fourth cycle (secondary schools), The National Curriculum Framework organizes mathematics education in two dimensions: mathematical processes and mathematics concepts.

Mathematical processes are given by the following list: 1. Presentation and communication, 2. Mathematical connections, 3. Logical thinking, argumentation, and deduction, 4. Problem solving and mathematical modelling, 5. Applying technology.

Document is written in terms of expected learning outcomes. As an example, the following table provides the learning outcomes for the domain Algebra and functions.

Domain	Learning outcomes
Algebra and functions	• establish, express, and anticipate numeric patterns in sequences that are defined by specific numbers, and analyse recursively-defined sequences and those defined by general numbers/terms (in particular, arithmetical and geometric progressions) • substitute formula variables with numerical values, calculate the value of the remaining variable[s], and express a variable in terms of all others • calculate with exponents, simple algebraic expressions, factorials, and binomial coefficients • describe and derive simple numerical relationships by using and performing mutual conversion between formulas, tables, graphs, and text; recognize, compare, and interpret these relationships • recognize, determine and interpret the characteristic elements and properties of simple functions; analyse linear, quadratic, exponential, logarithmic, and trigonometric functions; and use their properties • solve linear, quadratic, exponential, and logarithmic equations, inequalities, and equation systems by calculating in the set of real numbers, both graphically and with the help of a computer • apply functions and their graphs, equations, and inequalities when solving problems in mathematics, other educational areas, and their everyday lives.

Table 12. Examples of learning outcomes, National Curriculum Framework 2010, Croatia

Conclusion

Curricula in all partnering countries prescribe content to be covered by certain programmes and bring recommendations or guidelines on how topics could be presented, but with no strict rules concerning teaching methods, although

students' active learning strategies are officially promoted. Evidence from international studies PISA, TIMSS and TALIS point out that traditional transmissive practices predominate in everyday classes in many countries, which might act as a constraint for the implementation of IBMT. Teachers autonomously write their lesson plans which usually include information on learning goals for the lesson, learning methods, student activities and didactical material. Regarding textbooks, national policies vary concerning their approval by the national authorities, but in each partnering country great majority of teachers follows one or two main textbook series. For preparing a lesson, teachers also use other resources.

Except in the Netherlands, materials written to be used by inquiry-based approach are scarce and hardly available. This may indicate that there is a gap that could be filled by new materials designed in project MERIA providing teachers support in implementing IBMT. It is a requirement for these materials to be developed according to standards set by the most used textbooks.
Comparing the topics listed in the national curricula of partnering counties, we arrive at two categories of content areas. First category may serve as a source of topics that will easily fit in programmes of all partnering countries, while the second category provides ideas for additional (extracurricular) modules. In addition, we bring a description of mathematical processes that are encouraged to be used by some national curricula.

Content areas listed in all partnering countries	Content areas listed in some but not all partnering countries	Mathematical processes
Numbers Algebra Functions Calculus Geometry (including Trigonometry and Vectors) Probability and Statistics / Data	Sets and Logic Sequences and Series Differential Equations Combinatorics	Presentation and communication Mathematical connections Logical thinking, argumentation, and deduction Problem solving and mathematical modelling Applying technology Mathematical thinking Representing mathematical objects and situations

Table 13. Content areas and their presence in national curricula

The role of teachers in MERIA

The most important factor affecting student learning is the teacher and seemingly more can be done to improve education by improving the effectiveness of teachers than by any other single factor (Sanders et al., 1997). The Eurydice report further notes:

"Effective mathematics teaching depends to a large extent on the expertise of teachers; consequently, their knowledge of the subject – of mathematical principles and processes – and their professional training are crucial. Good teaching is reliant not only on teachers' mathematical subject knowledge and skills, but also on their understanding of how to teach their subject and of how students learn – both of which are essential if teachers are to reflect on and respond to the needs of their students." (Eurydice Network, 2011)

When referring to teachers' qualifications in project MERIA, we accept and emphasize the balance between teacher's subject knowledge and pedagogical knowledge following the academic research and institutional recommendations. To quote Eurydice report:

"…concerning highly qualified mathematics teachers, the US National Council of Teachers of Mathematics (NCTM, 2005) states that they must have 'an extensive knowledge of mathematics, including the specialized content knowledge specific to the work of teaching, as well as a knowledge of the mathematics curriculum and how students learn'. In other words, in addition to a 'profound understanding of fundamental mathematics' (Ma 1999, p. 19), teachers must also have what has been coined by Shulman (1986) as 'pedagogical content knowledge', i.e. the practical understanding of how to apply their knowledge and adapt it to their teaching, as well as 'curriculum knowledge', which refers to the knowledge of the content, materials and resources used for teaching, how they are arranged and ways to use them." (Eurydice Network, 2011).

Project MERIA proposes provision of support to mathematics teachers in reconsidering their teaching methods and designing high quality lessons. To ensure usability of designed materials and improve the experience of MERIA workshops the project team needs to understand needs and barriers of project's target group of teachers. As main demographic challenges of the teaching profession, OECD survey (2005) singles out the supply of good quality teachers, their ageing, the unequal gender distribution and the weak links between teacher education, teachers' professional development and school needs.

Strong support to teachers in the process of implementing (elements of) IBMT can be ensured by universities. Speaking in general about collaboration between higher educational institutions and high schools, former experiences among partners show that this collaboration is not intensive. This being said, there are interesting and productive initiatives. Let us mention two in **the Netherlands**. A first is the so-called VO-HO network, which translates as the secondary education – higher education network. A second example concerns regional networks of schools in the neighbourhood of a university. The most successful example of the latter is the U-Talent network in the Utrecht region, https://u-talent.nl/, in which over 30 schools collaborate with Utrecht universities in the field of teacher professional development, research, and innovative teaching approaches for students.

In **Denmark**, there are more forms of collaboration between high schools and higher education:

1) Some of the universities invite math teachers every year to various lectures in subjects related to the high school.
2) There are different programmes for visiting the universities with students especially in science, but also in mathematics.
3) In connection with the interdisciplinary examination assignment on the third year, the universities provide staff and laboratories for some students.
4) The Association of Math Teachers has a collaboration with the universities in developing courses, conferences and workshops for teachers.
5) CMU – Center for Computer based Math Teaching - offers continuing education for math teachers to target the use of computers in teaching for didactic purposes.

In **Croatia**, collaboration between high schools and higher educational institutions is mostly organized as ad-hoc collaboration depending on a local initiative. Only a few schools, like the MERIA partner XV. Gymnasium, have regulated collaboration with various faculties, departments or institutes.
PISA survey in 2009 confirms the shortage of good educators in some European countries by analysing the percentage of 15-year old students whose principals reported that their school's capacity to provide instruction was hindered by a lack of qualified mathematics teachers. For the partnering countries we can see strong variations. Although the result in the Netherlands (30%) is quite high, survey shows that Denmark (2%) and Slovenia (1%) are not at all struggling with the same problem of lack of mathematics teachers.
Croatia did not take part in that survey, but the 2016 recommendations of the Croatian Employment Service put the mathematics teacher profession on the list of deficient professions and recommends further investment in scholarships and entrance policies.

Conclusion

That *"effective mathematics teaching depends to a large extent on the expertise of teachers"* (Eurydice Network, 2011) is recognized in MERIA project which proposes substantial support to teachers in reconsidering their teaching methods and designing high quality lessons. Valuable feedback on everyday teaching practices, but also on challenges in implementation of IBMT has been provided by teachers from the associated schools. The obtained information contributes to needs and barriers analysis planned in the project.

Strong support to teachers in the process of implementing (elements of) IBMT can be ensured by universities. The focus of MERIA is to strengthen the connections between secondary mathematics education and universities, as well as to provide opportunities for professional development of teachers in the field of inquiry based teaching and other student-centred approaches. In respect to mentioned challenges, project MERIA is valuable as a meeting point for educational institutes, especially higher educational institutions and secondary schools. It provides an environment for educational experts with various background to collaborate and develop innovative teaching practices. As well, the project team strives to provide additional opportunities for upgrading teaching skills of in-service mathematics teachers. Accordingly, project MERIA will try to contribute by promoting profession of mathematics teachers and highlighting the importance of quality teachers for mathematics education.

Teachers' initial education

Initial education of teachers is closely related to the supply of qualified teachers. The requirements for the initial mathematics education are rooted in pedagogical research. Mathematics educators can be roughly divided into generalists (primary school) and specialists (secondary school or higher). In different countries initial education for these two groups may vary. As MERIA focuses on secondary education, let us shortly consider requirements for obtaining teaching qualifications in partnering countries for teachers in high schools.

"In EU countries participating in TIMSS 2007 international mathematics study, on average 93% of eighth grade students had teachers with a university degree. Only Slovenia is reported to have approximately half of students who had teachers who had completed post-secondary education but not university (Mullis et al. 2008, pp. 248-49)."

In **Slovenia**, all mathematics teachers in high school have the same level of qualification/degree required by the state (it differs only in programmes for teachers in upper secondary general education and upper secondary technical and vocational education). Qualifications can be obtained from different faculties:

- Faculty of Mathematics and Physics (University of Ljubljana)
- Faculty of Education (University of Ljubljana)
- Faculty of Natural Sciences and Mathematics (University of Maribor)
- Faculty of Education (University of Maribor)
- Faculty of Mathematics, Nat. Sciences and Information Technologies (University of Primorska).

On the other hand, in the **Netherlands** not all teachers in high schools have the same degree. For lower secondary (grades 7-9, 12-15 years old students), mathematics teachers have a bachelor's degree awarded by an institute for higher education (university of applied science). Entrance level is havo (5-year secondary education) and teacher education is organized by the education faculty. The teacher education is a four-year full-time programme. As a less frequent alternative, a bachelor's degree in mathematics, awarded from a university, may be accepted as well, if an educational minor is part of this three-year programme.
For higher secondary (grades 10-12, 16-18 years old students), teachers need a master's degree awarded by a university. Entrance level is vwo (the 6-year pre-university secondary education) and a bachelor's in mathematics. The master programme is a 1-year full-time programme or embedded in a 2 –year research master. It is part of the Graduate School of Teaching or based within the science faculty. As an alternative for a higher secondary degree, universities of applied sciences offer an educational master. Entrance requirements include a teaching degree for lower secondary, and the education consists of a two- or three-year part-time programme.
To get tenure as a high school teacher in Denmark one must have a university master's degree (300 ETCS) within the school disciplines (typically 2 disciplines, one of which is on bachelor level)). In order to teach mathematics this must include 120 ECTS specified as

- 60 ECTS core subject points including calculus, mathematical analysis, geometry, linear algebra, algebra, probability and statistics
- 30 ECTS in-depth points including modelling in science, advanced courses in core subjects (see above) and supplied with discrete mathematics, algorithms or dynamical systems
- 20 ECTS broadness points including history of mathematics, programming/IT and modelling in one or more subjects
- 10 ECTS-points including theory of science in mathematics and didactics of mathematics.

Adding to this, a teacher must complete a didactical course during the first or second year after employment at a high school. This course consists of theoretical

and practical didactics in each teaching subject finishing with one written assignment assessed by examiners. The practical part takes place with a supervisor from the employment school.
A report from August 2015 indicates, that an increasing part of the teachers only have the minimal 120 ECTS-points or even less (Eyrich Jessen, Holm, & Winsløw, 2015). This is due to lack of math teachers but means that a considerable amount of lessons is provided by teachers whose formal qualifications (i.e. from engineering, economy, primary school) are mediocre. Of course, some of these maybe excellent teachers but the situation as such is very worrying.

In **Croatia**, degree of mathematics teachers working in high schools is regulated by the national Law on education ("Zakon o obrazovanju"). According to the law, the required degree is the master's degree. The same law regulates the employment exception which can be considered if the tender cannot be realized by the person of adequate degree (by admitting e.g. a person with the bachelor's degree). A great majority of the mathematics teachers presently working in schools do have a master's degree in mathematics education. Degrees offered by Croatian universities are either master's degrees for the subject teacher (mathematics teacher) or two-subject teacher (mathematics and physics, mathematics and informatics teacher). For mathematics teachers of the second cycle of primary schools and for secondary schools, required degree is obtained at Department of Mathematics of University of Zagreb, University of Osijek, University of Rijeka and University of Split.
"National educational systems also differ in the way how areas of knowledge and skills for mathematics teaching are regulated. In some countries these areas are regulated centrally (on a national level), while in others higher education institutions have full autonomy in defining guidelines for mathematics educations. These are:

A. *Knowing and being able to teach the official mathematics curriculum*
B. *Creating a rich spectrum of teaching/learning situations and teaching materials*
C. *Developing and using a variety of assessment instruments for formative and summative purposes*
D. *Identifying and analysing students' learning as well as their beliefs and attitudes towards mathematics*
E. *Tackling students' difficulties in mathematics*
F. *Teaching mathematics in a gender sensitive way*
G. *Collaborating with peers, parents, authorities, etc.*
H. *Conducting research, alone or with colleagues and using research results in daily teaching practice."* (Eurydice Network, 2011)

Data from 2011 show that Netherlands has all these areas regulated centrally, Slovenia regulates all areas centrally except two (D. and F.), while Denmark

centrally regulates areas B, D, E, G. Although it is not our intention to discuss national policies, for MERIA it is important to be aware of the restrictions implied by central regulations. Moreover, all the proposed areas of knowledge and skills are considered as a possible part or point of focus of MERIA workshops.

Conclusion

Description of initial education for mathematics teachers in secondary schools guarantees the level of their knowledge and skills in all partnering countries. Teachers in *upper* secondary schools are all required to obtain the same degree (the master's degree in mathematics education), which could provide a satisfactory starting point in terms of their understanding of mathematics, and may be seen in favour of implementation of IBMT. Nonetheless, each of these systems allows for exemptions and there are clear differences how the exemptions are implemented in each country.

Teachers' professional development

Teachers' continuous professional development is an imperative of the profession:

"Following their initial education, mathematics teachers must continue to update their knowledge and skills. Opportunities for teachers to engage in continuing professional development (CPD) can have a substantial impact on their work, their achievement, skills and attitudes as well as on their performance and job satisfaction." (Villegas-Reimers, 2003).

One of the main aims of MERIA project is to provide support in implementing IBMT to mathematics teachers in partnering countries. To accomplish that MERIA team will design new materials that support IBME and organize a series of workshops for teachers in using student-centred methods and new materials, as well as how to design their own innovative materials and upgrade their teaching skills. Workshops will be planned and prepared in great details, which will be described in one of six project's results, called Workshop Guide.

"Teachers of mathematics must not only deliver the curriculum, but they must be able to adapt their teaching methods to the changing needs of students. They must learn to integrate new materials and technologies, and make use of research results relating to student learning and teaching practices in mathematics." (Smith, 2004).

To make MERIA workshops efficient, project team will focus its activities (module design and workshop themes) according to needs of teachers in the partnering countries. To begin with, we provide an overview of opportunities that teachers

have for professional development. The results on teachers' attendance of professional development courses are presented in the PISA 2015 report (concerning teachers of 15-year old students):

Country	Teachers attended PD in last 3 months (%)
The Netherlands	55.5
Denmark	41.2
Slovenia	46.8
Croatia	54.9
OECD	50.9

Table 14. Attendance of professional development courses (PISA 2015)

In **Denmark**, the offer includes that:

1) The professional consultant in mathematics provides on behalf of the Ministry meetings for teachers twice a year with the latest news in math teaching in upper secondary school according to the curriculum;
2) The association of Math teachers (Matematiklærerforeningen) develops and organizes various courses covering different needs in upper secondary school;
3) Universities offer continuing education to teachers who do not meet the official minimum requirements;
4) Continuing education to perform special tasks at the school in mathematics;
5) DASG – Danish Science High Schools (Danske Science Gymnasier) – a network of colleges that develop courses and materials for teachers in science and mathematics in high school.

There are different policies at the high schools, but most teachers have the opportunity for their professional development. However, following the collective agreement in 2013 the possibilities are in effect reduced quite a lot because of a greater workload and teachers are challenged finding time for professional development.

In **the Netherlands**, schools have a budget for professional development, which either can be spent according to school administration policies, or on individual teachers' or teacher teams' initiatives. Professional development courses are certified by a national organization, RegisterLeraar (https://www.registerleraar.nl/). Currently, plans are in preparation for a mandatory number of credits each teacher should obtain, but this is subject to debate. Professional development courses and workshops are organized and

delivered by different institutions, such as teacher training centres in universities and universities of applied science, the Dutch association of mathematics teachers, and commercial providers.

The intended programme of MERIA workshops will also fit with the national recommendations for professional development, as can be seen from the following two quotes:
"Knowledge and skills for mathematics teaching to be developed through CPD (Opportunities for teachers to engage in continuing professional development), as advocated by central authorities in **Slovenia**:
Knowledge and skills related to subject content

- Mathematics curriculum and curriculum reform
- Integrating mathematics with other subjects
- Improving students' critical thinking or problem-solving skills
- Applying mathematics in real-life contexts
- Communicating about mathematics

Knowledge and skills related to teaching methods

- Using assessment for formative purposes
- Integrating ICT into mathematics
- Detecting and tackling students' difficulties in mathematics
- Differentiating teaching for students with different abilities and motivation levels"

"In Slovenia, the Ministry of Education and Sport issues an annual catalogue of teacher professional development opportunities. Among the seminars offered is one on 'Mathematics through Research and Problem Solving'. In this seminar, teachers learn about the importance of problem-based knowledge, the transition from closed problems to open-ended problems and the planning and application of problems-based research to different stages of learning." (Eurydice Network, 2011, p.125)

In **Croatia**, professional development for teachers is mostly organized by the Croatian Education and Teacher Training Agency. It offers education on national and regional levels. On national level, two-day meetings are organized half-annually, for more than 300 teachers attending a meeting. Meetings provide a number of lectures usually on various topics. Details can be found on http://ettaedu.azoo.hr/ and on http://www.skole.hr/obrazovanje-i-tehnologija. Croatian Mathematical Society also offers opportunities for professional development. It organizes once a month "Mathematical evenings", with talks given by various presenters (mostly teachers). It also organizes the Congress of Mathematics Teachers biannually, with more than 700 math teachers attending.

In last decade, such meetings are also successfully organized by local mathematical societies, e.g. in Istria and in Osijek.
In developing MERIA workshops, project team will follow the needs analysis based on the available data and information provided by teachers participating in semi-structured interviews, but also general guidelines following European educational documents will be followed:

Timperley, Wilson, Barrar, & Fung, (2007) found that *"professional development was more effective when it went beyond generic pedagogy by providing teachers with a range of mathematics-based content and teaching methods that were specific and exclusive to mathematics. Elements that appeared to be particularly successful in terms of changing mathematics teaching in ways that led to positive student outcomes included those that engaged teachers in the current research findings that underpinned the particular methods being advocated. Moreover, they shared an emphasis on developing students' conceptual understanding of mathematics and encouraged multiple approaches to mathematical problem-solving."*
Although only the participants will be able to confirm the value of MERIA workshops, indicators that there is a greater demand for such workshops could be found also in the Eurydice report (Eurydice Network, 2011):

"In terms of actual CPD participation, data from TIMSS 2007 shows that on average in the participating EU countries… at eight grade… the proportion of students with teachers who had undergone professional development in improving students' critical thinking or problem-solving skills was relatively low with 31%", (Mullis, Martin, & Foy, 2008).

Despite these recommendations, dissemination of resources for IBMT and teachers' professional development that would have effect day-to-day teaching remained limited in Europe for years. Large-scale implementation of IBMT is a vision supported through various European projects (Maaß & Artigue, 2013), with project MERIA contributing to this goal too.

Conclusion

There is a variety of opportunities for professional development in partnering countries. The topics of meetings vary and do not follow a strict programme, neither they focus around a specific theme. The subject of inquiry based teaching has been present in workshops for professional development, but practical information or "hands-on experience" supporting teachers to implement such approach on their own is rare. MERIA workshops will focus on the practical side of using IBMT approach and provide teachers with materials and guidance in doing so. Consequently, it may happen that MERIA workshops have some small

overlaps with the existing offer for professional development, but it is nonetheless expected that this kind of workshops will provide a new and valuable experience for teachers in all partnering countries.

Assessment of mathematics achievements of high school students

Through MERIA the project team will provide support to teachers in enriching teaching from using traditional teaching methods only, towards more diverse approaches which are student-centred and supplemented by inquiry-based situations. The focus shifts to higher cognitive skills, like analysing, hypothesizing, proving, problem-solving, evaluating and modelling. An adaptation to appropriate assessment methods should follow also. Hence new materials designed within MERIA include guidelines for various methods of assessment especially assessment giving feedback both to students and teachers if the expected target knowledge has been reached.

Assessment is an important part of a curriculum cycle. It highly influences both teachers' approaches and organization of time dedicated for development of different knowledge and skills as well as students' studying and learning. Thus it provides an opportunity to influence students' learning outcomes in a desired way, as it is proposed in so-called constructive alignment of teaching and learning activities and assessment (Biggs, 1996).

In this section we first review requirements that high-school students meet at the end of the upper secondary educational cycle, mostly in the form of external assessment, and then also assessment forms present in schools.

In the **Netherlands**, both for the 5-year secondary stream (havo) and the 6-year stream (vwo), final assessment consists of two parts: a 3-hour national written examination, which determines 50% of the students' final grade, and school-based assessments, which determine the other 50%. For the latter, schools are free to organize their assessments. In most schools, it encompasses different tests throughout the final year, and may range from a format similar to the national examination to computer activities or writing small theses.

For assessment methods in general there are also some rules, but only to a limited extend. For mathematics A at havo, it is mandatory to have at least one test involving the use of ICT tools on the domain of statistics. More information is available on the web page:
http://www.slo.nl/downloads/documenten/Brochure_20Vernieuwing_20examenprogrammas_20wiskunde_20havo-vwo.pdf/.

In regular school practice teachers normally present a test to their students at the end of each textbook chapter. Textbook publishers provide examples tests that can be adapted by the teacher. During the work on a chapter, teachers also often provide a small (formative) assignments to their students.

In **Denmark**, each of the three levels (A, B, C) has a different type of the graduation exam. The students are not taking exams in every subject. They have a total of 9 exams in 3 years. The students are informed in media if they are obliged to take an oral exam, written exam, both or (in lower years) none.
Written exams are taken only on A and B level and they include tasks to be solved on paper without using any materials as well as tasks to be solved using computer algebra system and textbooks. There are strict rules for organizing the written exam. The schools have exact time and date for the exam and all the students will have exactly the same conditions and will be solving the same tasks all over the country. All responses are assessed by two external examiners.
For oral exam, the students receive the questions for the examination before the course ends and at the exam they present an answer to one question selected from the pool by chance. The oral examinations are less restricted, but the questions must be approved by the examiner before the exam to secure that the questions meet the prescribed level, cover the curriculum requirements and are formulated in a clear language.
The described exam forms are based on the STX programme. At HHX there a similar structure, but HTX has mathematics only A- and B-levels and their exams are different. However, still there is an oral and a written exam.
From 2017 there are some changes, but exams are basically the same. The oral exam at C- and B-level for STX will have a session before the individual exams described above, where the students in pairs work with a new and unknown open issue (C-level 90 minutes, B-level 120 minutes).
Assessment forms present in Denmark's schools during the year include: students who will continue with math after first year (B- or A-level) will have a compulsory test at the end of year similar to the written exam. From 2017 there will for B-level students be an oral test at the end of the year. Also, in March the year they will finish the subject, they will have a compulsory test. During the year the students have assignments. The number depends on the teacher, but the law demands a certain time of homework for the students e.g. at B-level they will have to deliver 45 hours of homework with assignments during the year. It could be 20 assignments of 2 hours each. If the teacher wants to make tests during the year after a completed topic he is free to do so. It could be like the written exam or the oral. But assessment is ongoing throughout the year with grades given 3 times a year. There is no requirement for the feedback given during the year, but the formative evaluation is recommended for the ongoing evaluation.

In **Slovenia**, the final exam is an external exam which takes different form in Upper secondary general education (gimnazija) and Upper secondary technical education. Students in Upper secondary general education (gimnazija) take

matura as the form of graduation exam. It consists of 5 subject and mathematics is compulsory. Students can decide whether to take the basic level or advanced level of the exam. 'The curriculum contents for both are the same, but required standards differ. The written test for the advanced level, in addition to compulsory items for the basic level, contains additional advanced level items. For oral examinations, the expected knowledge for the advanced level is specified in the *matura* standards (i.e. theoretical explanation of the definition of a limit versus the calculation of the limit only). Students receive grades from 1 to 5 for the *matura* exam at the basic level and from 1 to 8 at the advanced level. The sum of grades from all five *matura* subjects is used as a criterion for entrance to tertiary-level education programmes with a limit on the number of new students' (TIMSS Advanced, 2015).

Students in Upper secondary technical education take *vocational matura* as the form of graduation exam. Vocational matura is also the form of graduation exam for students who successfully finish Upper secondary vocational education programme and after that Vocational-technical education programme. Vocational matura enables access to professional types of higher education (vocational colleges and colleges but not university courses). It consists of four subjects, of which the third subject can be chosen between foreign language and mathematics. Written part of the exam encompasses 11 shorter questions and 3 structured questions, out of them student chooses only two. Oral part of the exam is carried out in a school and the questions are in the form of a situation (based on real life or profession: economy, engineering, electronic, medical, tourism, gastronomy, agriculture …) and situation-related questions. Computer or graphing calculator (if used during instruction in the classroom) are allowed on the oral part exam.

There is also some national research on assessment in Slovenia (Žakelj (Eds.), 2013). For example, the role in national examinations of algebra and of mathematical thinking, two foci of the new upper secondary curricula, has been investigated. There are several researches on external examinations like general matura, vocational matura and National Examinations in Basic Schools and some other researches on math education but there is no large-scale national research on assessment.

The field of assessment in Slovenia's schools is regulated by the Rules on Assessment of Knowledge in Upper Secondary Schools in form of general rules for assessment. More detailed rules are specified by teachers within a group of math teachers in every school. Some guidelines related to assessment are also included in the math curriculum, like in curriculum for Upper secondary general education:

- Knowledge should be assessed according to the goals and should be

assessed regularly.
- Various forms of assessment should be used, so the students have the opportunity to show most of their knowledge and skills.
- Students should get at least four written grades and at least one oral grade per year.
- Students should be responsible for their learning.
- For oral assessment, knowledge that cannot effectively be assessed by written tests should be assessed mainly (the use of strategies, problem solving, understanding of concepts and procedures, communicating). Enough time should be devoted to oral assessment.

In the math curriculum for Upper secondary technical education different forms of assessment are planned, besides written tests and oral assessment also mathematical investigation, empirical investigation and project paper. The number of grades for different forms of assessment is not prescribed.

In Croatia, the state matura exam is an obligatory exam for gymnasium students providing them with the leaving school certificate. It is also required for other secondary students pursuing further higher education. Mathematics is an obligatory subject and is offered on two levels of achievement, higher and standard level. Students choose one of these assessment levels in the winter of their last school year. Approximately half of the gymnasium students participates at higher level of mathematics exam. A significant number of vocational students, in particular of technical orientation, chooses to take the state matura mathematics exam at the higher level, in order to continue their study in higher education in direction of science, technical and biotechnological studies. Vocational students who do not pursue higher education, do not have to take state matura exams.
For assessment in schools there are quite restrictive rules regulated by the Croatian Education and Teacher Training Agency, prescribing number and format of summative types of assessment in all subjects per week. Recently some recommendations on other formats of assessment has been promoted (especially, formative assessment) at meetings for professional development for teachers organized by the Croatian Education and Teacher Training Agency and Croatian Mathematical Society.

Conclusion

Assessment regulations in all partnering countries have been viewed in their major aspects: the role of external summative assessment in the form of the graduation (final) exam, the role of assessment in school, and the extent to which formative assessment is regulated. For MERIA, we may derive two important conclusions. First, the project team must be aware of the strong barrier to implementation of IBMT - limited time that teachers are willing to “sacrifice” for

skills not tested in the external final summative exam. Second, the need for evidence of students' progress during "research-like" activities is being discussed. The project will try (along with new materials) to provide teachers with formative assessment methods that will give feedback on student's progress in attaining the target knowledge expected in the scenarios.

References

Biggs, J. (1996). Enhancing teaching through constructive alignment. *Higher Education*, *32*, 347–364. Retrieved from https://pdfs.semanticscholar.org/5912/ba605d0baba1b6af864dd4949b3b3ceb995e.pdf

Bowen, G. A. (2009). Document Analysis as a Qualitative Research Method. *Qualitative Research Journal*, *9*(2), 27–40. https://doi.org/10.3316/QRJ0902027

Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Dordrecht: Kluwer.

Bruder, R., & Prescott, A. (2013). Research evidence on the benefits of IBL. *ZDM - The International Journal on Mathematics Education (Zentralblatt Für Didaktik Der Mathematik)*, *45*(6), 811–822.

CITO. Resultaten PISA-2015 in vogelvlucht. Retrieved from https://www.rijksoverheid.nl/documenten/rapporten/2016/12/06/resultaten-pisa-2015-in-vogelvlucht

College voor Toetsen en Examens. (2018). Examenblad. Retrieved from https://www.examenblad.nl/

Dorier, J.-L., & Garcia, F.-J. (2013). Challenges and opportunities for the implementation of inquiry-based learning in day-to-day teaching. *ZDM - The International Journal on Mathematics Education (Zentralblatt Für Didaktik Der Mathematik)*, *45*, 837–849.

Eurydice Network. (2011). Mathematics Education in Europe: Common Challenges and National Policies. https://doi.org/10.2797/72660

Eyrich Jessen, B., Holm, C., & Winsløw, C. (2015). *Matematikudregningen*. *Institut for Naturfagenes Didaktik, Københavns Universitet*. Retrieved from http://www.ind.ku.dk/projekter/matematikudredning/Matematikudredningen2015endelig.pdf

Godino, J. D., Batanero, C., Canadas, G., & Contreras, J. M. (2015). Linking inquiry and transmission in teaching and learning mathematics. In K. Krainer & N. Vondrova (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 2642–2648). Prague,.

Gravemeijer, K., Bruin-Muurling, G., Kraemer, J.-M., & van Stiphout, I. (2016). Shortcomings of Mathematics Education Reform in The Netherlands: A Paradigm Case? *Mathematical Thinking and Learning*, *18*(1), 25–44.

Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. (A. Routledge, Ed.).

Heuvel-Panhuizen, M. Van den. (2000). Mathematics education in the Netherlands. *Freudenthal Institute CD-Rom for ICME9*. Retrieved from www.fisme.science.uu.nl/staff/marjah/documents/TOURdef+ref.pdf

Labuschagne, A. (2003). *Qualitative research: Airy fairy or fundamental? The Qualitative Report*. Retrieved from http://nsuworks.nova.edu/cgi/viewcontent.cgi?article=1901&context=tqr

Maaß, K., & Artigue, M. (2013). Implementation of inquiry-based learning in day-to-day teaching: a synthesis. *ZDM - The International Journal on Mathematics Education (Zentralblatt Für Didaktik Der Mathematik)- Mathematics Education*, *45*, 779–795.

Mullis, I.V.S, Martin, M. O., & Foy, P. (2008). *TIMSS 2007 International Mathematics Report: Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades.*

Nuffic. (2015). Dutch education system. Retrieved from https://www.epnuffic.nl/en/study-and-work-in-holland/dutch-education-system

OECD. (2016). *Ten Questions for Mathematics Teachers… and How PISA Can Help Answer Them*. OECD Publishing. https://doi.org/10.1787/9789264265387-en

OECD. (2018). *PISA 2015 Results in Focus*. OECD Publishing. https://doi.org/10.1787/9789264266490-en

Sanders, W. L., Wright, S. P., & Horn, S. P. (1997). Teacher and classroom context effects on student achievement: Implications for teacher evaluation. *Journal of Personnel Evaluation in Education*, *11*, 57–67.

Smith, A. (2004). *Making mathematics count : the report of Professor Adrian Smith's inquiry into post-14 mathematics education*. Retrieved from http://dera.ioe.ac.uk/id/eprint/4873

Steele, M. M. (2005). Teaching Students With Learning Disabilities: Constructivism Or Behaviorism? *Current Issues in Education (on-Line)*, *8*(10). Retrieved from http://cie.ed.asu.edu/volume8/number10

Timperley, H., Wilson, A., Barrar, H., & Fung, I. (2007). *Teacher professional learning and development: Best evidence synthesis iteration (BES).*

TIMSS Advanced. (2015). *Description of the Advanced Mathematics Programs and Curriculum*. Retrieved from http://timssandpirls.bc.edu/timss2015/international-results/advanced/timss-advanced-2015/mathematics/programs-and-curriculum/slovenia-description-of-advanced-mathematics-programs-and-curricula/

TIMSS Encyclopedia. (2015). Retrieved from http://timssandpirls.bc.edu/timss2015/encyclopedia/countries/netherlands/the-mathematics-curriculum-in-primary-and-lower-secondary-grades/

Villegas-Reimers, E. (2003). Teacher Professional Development: An International Review of the Literature. *Paris: UNESCO International Institute for Educational Planning.*

Žakelj, A. in ostali (Ed.). (2013). Posodobitev kurikularnega procesa na osnovnih šolah in gimnazijah: zbornik prispevkov zaključne konference in predstavitev predmetno razvojnih skupin. Ljubljana. Retrieved from http://www.zrss.si/digitalnaknjiznica/PKP-zbornik-prispevkov-zakl-konf/#/1/

Zakon o obrazovanju. (n.d.). Retrieved from https://www.zakon.hr/z/317/Zakon-o-odgoju-i-obrazovanju-u-osnovnoj-i-srednjoj-školi

Semi-structural interviews with teachers in associated schools

Plan of the research and questions for the first round of interviews

Prepared by: Blaženka Divjak, Faculty of organization and informatics, University of Zagreb, Croatia

Preliminaries

We have planned two interviews with teachers, one at the beginning and one at the end of the project. The interview will be performed with the teachers in selected schools. In each selected school, there will be 2-3 teachers of mathematics with whom we are going to work intensively. At the beginning of the project, we will interview teachers in order to set up the reference point (situation as it is). At the end we will have another interview with, hopefully, the same teachers to record the progress – if any. The interview at the end will be slightly different from the one at the beginning. The plan is to collect between 10 and 14 interviews.

In this paper we are going to set purpose, hypotheses and plan for the research with the emphasis on the first interview and phases that follow in the first six months.

Purpose and actors of semi-structured interview

Purpose: to gather data from teachers at the beginning of the project and towards the end; to formulate hypotheses about situation in the schools related to inquiry based mathematics teaching (IBMT) but also to get ideas about how to proceed with trainings and material design.

Actors. Interviewer: MERIA representative and Respondent: teacher

Why interview? Because it is a flexible tool for data collection, enabling multi-sensory channels to be used: verbal, non-verbal, spoken and heard (Cohen et al., 2007). We will use **semi-structural interviews** (interview guide approach) where topics and issues are covered and specified in advance in outline, but questions are open-ended and allow an interviewer to ask additional sub-questions in order to clarify or verify answers or to follow an interesting route opened by a respondent. The approach is predominantly qualitative and seeks for understanding because we will have a rather small sample (usually 4-6 interviews per country) and we are not aiming at generalizations of our findings. Despite of that we aim at structuring our findings as much as we can and probably compare results between countries.

Research hypothesis and questions

Hypothesis are written in a way to reflect our expectation and to allow testing (probably not in full rigor) according to the variables and constructs, we plan to introduce in the interview. Research questions are formulated in order to collect significant information that is important for the planning of other (intellectual) outputs of the project (training, scenarios, modules, conference etc.).

If there will be additional interests from the partners this research can be extended to meet scientific requirements (sample, reliability, validity, generalization power).

Plan of interview-based research

We will follow eight standard phases described below:

1. **Thematizing**. The prepared discussion paper was sent to the partners a week before kick-off meeting. Based on that, discussion was opened at the meeting and first reactions were collected. After that online discussion followed based on the document on Google Docs. In that phase, we agreed on the purpose and the general idea of the interview with teachers in a particular time frame and general domains to be covered.

2. **Designing**. In this phase, we need to translate purpose and research objectives into questions. Questions must reflect what we would like to find out of it. Specifically, we need to write down variables we need to assess (measure).

   Hypothesis for the first interview:
   H1a. Teachers are aware of IBMT approach and appreciate it but they do not implement[4] it in classroom practice of mathematics teaching.
   H1b. Teachers are aware of learning outcomes based approach and appreciate it but they do not implement it in classroom practice of mathematics teaching.
   RQ1: What other concepts/approaches they know and apply?
   H2a. Lack of adequate system support, teaching and learning material, teacher training and overloaded prescribed program are main reasons for not applying IBMT approach in teaching on regular bases.
   H2b. Lack of adequate teaching and learning material, teacher training and overloaded program are main reasons for not applying LO in teaching on regular bases.
   RQ2: What other obstacles for not innovating teaching and learning process can be recognized?
   RQ3: What useful ideas teachers have that enable IBME?
   RQ4: What kind of training and material teachers are interested in?
   H3. Assessment of students is not aligned with declared approach (concept) and teaching method.
   RQ5: What interesting assessment methods teachers use?
   RQ6: Are teachers interested in assessment method training?
   H4. Teachers have a positive attitude towards changes in teaching and learning and have concreate ideas what and how to change on the macro (system) and micro (classroom) level.
   H5. Teachers feel that IBME brings some additional positive outcome compared to the usual approaches and that it is worth the effort.
   RQ7: What ideas teaches have for a change on a classroom level?

[4] In blue color we try to note the same construct that are measured with several variables. In order to evaluate "implementation of IBMT and LO" we have to observe the set of the teaching materials, teaching methods and assessment methods used.

Domain	Variables	Questions	Prompts and probes	Related to H's and RQ's
Teacher's understanding of IBMT approach	1. Familiarity with IBME concepts and approaches 2. Familiarity with other concepts	What approach math teaching you find useful? Are you familiar with IBMT approach? What is your interpretation of IBMT? If you are familiar with IBMT and not implement it what is the reason for that? What other approaches you find useful?	Interviewer explain what "approach" is and give some examples if teacher ask for it. For example: RME, problem-solving	H1a H5 R1
Teacher's understanding of learning outcomes	1. Familiarity with learning outcomes 2. Familiarity with other concepts	What concept of math teaching you find useful? Are you familiar with LO and concept? What is your interpretation of LO? If you are familiar with LO concept and not implement it what is the reason for that?	Interviewer explain what "concept" is and give some examples if teacher ask for it. For example: definition and example of LO	H1b R1
Typical schedule or framework of an *ordinary* maths class (1 hour)	1. Typical schedule for one lesson 2. Type of teacher Preparation 3. Teaching method	Describe a typical schedule or schedules for classroom delivery. How you prepare for a class? What teaching method you used for different class delivery?	Probably you have a class with different aims – for example, when you mainly take new concepts; or when you assess your students' achievements…	H1a, H1b H2a, H2b RQ2

Exercises and problems sources and how teachers adopt them	1. Material for preparation for teachers 2. Material for preparation for teachers 3. Teacher prepare material for herself	What material (books, notes, online resources…) you use? What material you ask your students to use? Do you collect good problems from different sources, modify material, prepare your own problems, share with other teaches?	Are there textbooks you and your students use? Are you satisfy with them? Do you participate physically or online in communities of practice with other teachers? What online resource you find useful?	H2a, H2b RQ2 RQ4
Training available and used by teachers	1. Availability of teacher training 2. Attendance to training 3. Usefulness of training	Are there teacher training available? Do you attend? Do you find offer of teacher training useful?	Why do you attend or not attend available teacher training? What topic you like to cover?	H2a, H2b RQ3 RQ4
Assessment of students, especially their problem solving skills	1. Assessment methods used 2. Assessment method used for problem solving	What is the most often methods you use to assess students? What are the reasons you use that methods? How are you sure that your students acquire problem solving skills or higher order cognitive skills?	Interviewer can explain (if asked or if he feels that a teacher does not understand the term) what assessment method is or higher order cognitive skills. Do you need any additional training on assessment methods?	H1a, H1b H3 RQ5 RQ6

Teacher attitude towards changes in teaching methods	1. Teachers' attitude towards changes in education in general 2. Teachers' attitude towards changes in math education 3. Teachers' ideas about concrete changes in math education	Do you feel that changes in our national education is necessary? What change, if any, should be introduced in teaching and learning mathematics? What exactly you would change in your classroom? Why not immediately?	Are you satisfied how our national system is organized? If you are able (have first-hand experience or theoretical knowledge) compare our system with some others. Are you satisfied with results of students in your class?	H4 H5 RQ6 RQ7

Table 1. Summary of main domains, questions, some prompts and probes and their relation with hypotheses and research questions.

Before starting with interviews it should be decided who is going to be interviewer, there should be a training for them (online), and detailed guidelines, forms and equipment to be used. The guidelines will also cover privacy and other ethical issues.
The *consent form* should be prepare in advance together with one-pager information about the MERIA project. These will be sent to the selected teachers beforehand.

3. **Interviewing**. At the beginning, the interviewer should ask a respondent (interviewee) if she/he needs additional information about the MERIA project and purpose of interview besides information that has been sent before. Then the interviewer informs the respondent about timeframe for interview, privacy and rights of the respondent as well as the use of data. Finally, the interviewer asks the respondent to sign the consent form to use the (anonymised) data and to record the interview.

The interviewer asks the teacher to feel out simple form with background information.

Her/his degree and university that issued it	Years of experience in math teaching	Years of experience in teaching in this school	Classes you teaching	Statues (e.g. beginner, mentor, advisor)	Gender

Table 2. Form for general information

The interviewer starts with interview questions according to Table 1. The interviewer asks the teacher if she would like to add something or to ask some the interviewer clarification about handling with collected data. The interviewer thanks the teachers.
Estimated time per an interview: 30 minutes.
Technical equipment needed: audio recorder to record interview, computer and software for analysis.

4. **Transcription**. It can be performed in several ways. Usually there is a transcript and/or a report available after interview in a country language. It is also very interesting to capture non-verbal communication, but we do not have resources for that. We ask the interviewer to prepare short report about the interview: was it according to plan? Were there any deviations? Were there any interesting ideas or loose ends that we can follow? Is there anything to ask in the second interview (at the end of the project)?

5. **Analysing**. Since we will have semi-structured qualitative interview with predominantly open-ended questions data analysis will inevitably be interpretative. It means that we will have reflexive, reactive interaction between researchers and the data that take into account social encounter in interpretation (Cohen et al, 2007). Therefore, we have balance holistic view on data with atomised data obtained out of coding and maybe scoring. It means that we according to (Cohen et al, 2007) take several stages in analysis:
    a. Generating natural units of meaning.
    b. Classifying, categorizing and ordering these units of meaning.
    c. Structuring narratives to describe the interview content.
    d. Interpreting the interview data.

    Coding means translation of questions responses and respondent information to specific categories for the purpose of analysis. Our attempt (Table 1) is to pre-code the questions and possible answers can be classified according to that. (*More has to be done on the topic of coding and decide what to cover.*)
    Content analysis is based on the testing of hypothesis.
    The scope of analysis should be further discussed (Skype meeting) because we can take at least two different approaches. First, let each country report on answers of the above questions with illustrative and underpinning quotes by teachers and produce one general report out of these country reports. Secondly, we can make more coherent analysis and comparison between countries and produce research paper on that, but this work cannot be fully covered under the scope of project MERIA.

6. **Verifying**. Here we must evaluate the issues of reliability, validity and generalizability of data from interviews. Verifying starts from the beginning of planning of the interview to ensure sound theoretical approach. However, our main aim is not to prepare rigorous research for publishing (even it can be desired side result), but to preform situation analysis for further project activities. We could try to involve master-students and use MERIA as a research project for them, but we have to discuss that possibility further as well as is it feasible to get valid results.

7. **Reporting**. There can be tree types of reports: report for partners, report for EC (if needed) and research paper. Report is going to be prepared firstly for project partners, then EC and finally for broader professional and scientific audience in form of research paper.

Note: Some of the research question and hypothesis can be used more frequently during project. For example, H4 and H5 after each workshop or other training with teachers.

Literature

1. MERIA project application form.
2. Cohen, L., Manion, L., Morrison, K. (2007). *Research Methods in Education.* Rutlenge Taylor Frances Group. London and New York.

Final note: The 3rd draft is prepared by Blaženka Divjak, FOI University of Zagreb in Varaždin February 19, 2017 based on the Draft of O2 & O3 description and comments given by partner and comments given by Mojca, Jeanette, Željka, Eva, Rogier and Matija.

Questions for the second round of semi-structural interviews with teachers in associated schools

Prepared by: Rogier Bos, University of Utrecht, the Netherlands

After analysing the first round of interviews, we have set to evaluate the following three *goals* for the second round of interviews:

I. Project impact on teachers in selected schools.
II. Satisfaction of participants with workshops and the final conference.
III. Quality assessment of developed teaching and learning material.

The questions for the second round of interviews are organized into six domains and the

Questions for the second interview

Domain	Questions	Related to H's, RQ's and goals
Inspiration from project MERIA	- What is the most important thing you take on board from the MERIA project? - What other ideas and activities do you take from the MERIA project? - What activities from the MERIA project inspired you? How? - Would you advise your colleagues to participate in the MERIA-project (or a similar one)? Why?	H5 I
MERIA activities	- What were your experiences at the MERIA workshops? - What were your experiences at the MERIA Final Conference? - Through the MERIA activities, did you find the sort of training/experience you were looking for? - Did the MERIA activities support you in bringing IBMT to practice?	H2 RQ4 II
MERIA outputs	- Did you read/use the Practical Guide or are you planning to? Why? - Did you read/use the scenarios or are you planning to? Which ones? Why? - Did you read/use the modules or are you planning to? Why? - Through the MERIA project, did you find teaching materials you were looking for? How? - Did the MERIA outputs support you in bringing IBMT to practice?	H2 RQ4 III

Implementation	- What taken from the MERIA-project have you implemented in your lessons? - Did that work? Did it bring what you hoped for? - Did you experience any obstacles in the implementation? - How did your students respond to your MERIA-related implementations? - Did you experience you had time and space in the school program to try IBMT-lessons?	H2 RQ2 III
Theoretical knowledge	- What do you know about IBMT (inquiry based mathematics teaching)? - What have you learned about IBMT from participating in the MERIA project? - What have you learned about TDS (theory of didactical situations)? - What have you learned about RME (realistic mathematics education)? - Has the MERIA-project increased your teaching repertoire/toolbox, in particular with respect to IBMT?	H1 I
Change	- Has the extent to which IBMT plays a role in your teaching changed structurally/consistently? - Do you think there should be more IBMT in the curriculum (national level), in your school, in your own classroom practices? - Why? - To facilitate more IBMT, do you think there is need for a structural change? - What changes?	H4 RQ7 I

Table 1. Questions for the interviews with teachers in selected schools at the end of the project

Inquiry-based teaching in partnering countries

Report on the first semi-structural interviews

Introduction

Interviews were held during May 2017 in 4 schools in Croatia, 3 in Slovenia, 3 in Netherlands and 3 in Denmark. In total, there were 31 teachers interviewed. Teachers were interviewed on the premises of their school or via Skype. Teachers have been informed about the purpose of the interview and at the beginning of the interview gave their consent to being recorded.
Interviewees teach mathematics in different programs. Interviewed teachers' length of service varied from 2 to 26 years.
All interviews were recorded and based on the data, interviewers wrote national reports. National reports were gathered and discussed in terms of conclusions for the project. We first present the national reports and then the conclusions on the level of project. Analysis of answers for the Slovenian, Dutch and Croatian national report is summarized with respect to the hypotheses and research questions, while the Danish national report and the conclusions are written with respect to domains.

Denmark

Prepared by: Jeanette Axelsen, Vordingborg Gymnasium, Denmark

Teachers' background

There are only two teachers with mathematics as their main subject. The others have physics, computer science or chemistry as their main subject. The gender, the teaching experience and the age of teachers vary. The experience varies from a few years to about twenty years. Their age varies between 30 to 50. Three of the teachers are working in the general high school called STX, two of the teachers are working in the high school of trade and economics HHX, and the last three are working in the high school with focus on science like engineering HTX.

Teacher's understanding IBMT approach

The teachers have no familiarity with the IBME-concept. Some of them have heard about it, but without knowing the exact content. Some of the teachers with physics as their main subject have been working with the inquiry based approach in physics. The experience is, that it is highly motivating for the students working like this. Most of the teachers are well familiar with the inductive approach to math using experiments by means of CAS, but it is with high management from the teacher. When asked about IBME, they think this is the same as this described approach.

Teacher's understanding of learning outcomes

Most teachers are familiar with i.e. cooperative learning as a concept, but it is only used by some of them and for the variety in teaching. Teaching with focus on the student's activity are used by them all, but in various forms. Some of them work with competences in a broader sense, where students should reflect on their own processes and learning.

Schedule or framework of an ordinary maths class

Most of the teachers have the feeling, that students like the same structure in the math lessons and are motivated working that way. That math invites to some particular form of work.

A structure often used is

- Introduction to the lesson
- Repetition of last times concepts
- Introduction to new concepts and methods
- Students do exercises using the new concepts and methods
- Gathering the exercises and the lesson

Lessons with proofs, proof exercises or experiments will be structured otherwise. The teacher's preparation will often be based on textbooks where the form and content of the lesson will take into account where students typically find the substance difficult. Didactical approaches are not a deliberate part of the preparation. It is at most variation in the teaching that is thought of.

Exercises and problem sources and how teachers adopt them

Most teachers have a textbook for the students. They can choose for themselves which one they like to use but is limited to the books available at the school. If many teachers want the same textbook for their class, they will have to reach agreement among themselves.
Most of the teachers prepare material for themselves, but it is very different how much they make themselves from scratch. Lots of materials are to be found at websites or they share with colleagues at school.
None of the interviewed have to use some specific materials for their teaching.

Training available and used by teachers

All the teachers in question have the opportunity to attend a course and to be supported from the school in the form of expenses for the course. However, no extra time are given unless you have a contract for a further continuing education program. Most of the teachers find it difficult to find the time to join courses though it could be relevant and interesting and useful. So, it is very much up to the teachers. It varies what kind of courses the teachers find useful, but they would all like to have some more didactical inputs.

Assessment of students

The assessment methods used is divided into two parts.
Test are used by every teacher to train the students in the form and with the content similar to the written exam at the end of the year. Tests like these are given at the end of a course for assessment. Also, projects with a report are used.

For the oral exam different kind of methods is used for assessment. When the students are working with exercises the teachers take a look and have a dialogue about the work. Also, small presentations in groups or with groups in front of the class are used. So, the assessment is ongoing.

The teachers are mostly testing for the exam, but they know it is not enough to learn math. Time and students' motivation to achieve good results are factors that make the choice fall to the exam-similar tests.

Teacher attitude towards changes in teaching methods

There is great willingness among the teachers to change their teaching practice. Most of them are constantly working to change their practice, both the small changes, but also the greater, especially at the school level.
In Denmark there is a new reform with the first students starting in August 2017. So, lots of changes are to come especially in the contents and didactics of the curriculum for mathematics.

Slovenia

Prepared by: Mojca Suban, National Educational Institute, Slovenia

H1a. Teachers are aware of IBME approach and appreciate it but they do not implement it in classroom practice of mathematics teaching.

Not supported. Half of the teachers are not familiar with the IBME approach, while the other half of the teachers have heard a bit about it and have tried to implement something similar in practice, but are not entirely sure they did it properly. They think that the IBME approach is similar to inquiry-based learning. Secondary school students like to conduct research and reach conclusions on their own. Students are more active if they are solving real-world examples (not just learning contents, but examples from real life).

H1b. Teachers are aware of learning outcomes (LO) based approach and appreciate it but they do not implement it in classroom practice of mathematics teaching.

Partly supported. Only one of the six teachers is familiar with the concept of learning outcomes, and says that she has to know what the students are required to know, and that this is important to her in the planning, implementation and assessment phase. Others say that they are not familiar with this concept and assume that it is the same as the expected achievements that they use for planning lessons (e.g. for preparing tasks). Secondary school students are familiar with the minimum knowledge standards or the end-of-programme learning outcomes, which they ought to achieve at the end of the school year. One teacher also recognised learning outcomes in formative assessment (the latter contains learning outcomes and information about how students' knowledge is progressing).

RQ1: What other concepts teachers know and apply?

They mention the following approaches as useful: problem-solving approaches, learning to learn, modelling, independent learning, working in pairs, cross-curricular integration, use of concrete examples to understand concepts, etc.

The teachers mostly carry out "traditional lessons" – by explaining new contents in front of the class. All of them mentioned the use of ICT, e.g. the use of interactive textbooks, online classrooms – Moodle, making applets – web animations, etc.

H2a. Lack of adequate system support, teaching and learning material, teacher training and overloaded prescribed program are main reasons for not applying IBME approach in teaching on regular bases.

Supported. The teachers who are familiar with IBME are aware of the advantages of this approach and appreciate it, but rarely implement it in practice. The reason for this is that they do not know enough about this approach and that they are overburdened by the prescribed learning programme. Teachers are only preparing the students for the matura examination. They need teaching and learning material, and training in this approach.

H2b. Lack of adequate teaching and learning material, teacher training and overloaded program are main reasons for not applying LO concepts in teaching on regular bases.

Not supported. Teachers know the concept of learning outcomes under the name expected achievements/results (five teachers) and use it for planning and implementing lessons.

RQ2: What other obstacles for not innovating teaching and learning process can be recognized?

Teachers are of the opinion that our education system needs to change. They have reported the following: "Changes are necessary, because the school is too efficacy-oriented; the teacher is too limited by the curriculum and pressured by the matura examination; due to time constraints, secondary school students cannot perform inquiry-based tasks, which are the most beneficial." It bothers them that when enrolling in general secondary school, students overestimate themselves and then limits have to be lowered. A novice teacher is of the opinion that changes are needed at the level of undergraduate education, because the faculty does not provide sufficient knowledge and they are not in touch with the actual conditions in school.

They wish for changes at the curriculum level, for the curriculum to be modernised, and for cross-curricular links to be written down more specifically. Teachers cannot decide whether to eliminate specific contents from the curriculum, because they find all of them important. Most of the teachers want a greater number of Mathematics lessons, but one of them thinks that is not the solution. They wish for greater autonomy in assessment; they want to be rewarded or at least commended more often; and want quality didactic aids.

RQ3: What useful ideas do teachers have that enable IBME?

They want to use more ICT while teaching and give more meaning to mathematical contents. They wish they were able to motivate secondary school students better; that they would possess more knowledge of psychology in the field of students with special needs; they wish they had more time; and that they

did not have to assess students. The reason why they are not introducing changes right away is that they have too little time and too many other tasks (substituting, projects, website, chronicle). They wish for as many ideas as possible, new knowledge, approaches to lessons, and a teacher handbook (cross-curricular links).

RQ4: What kind of training and material teachers are interested in?

Teachers use handbooks, textbooks, e-textbooks, e-materials, the Internet, and the magazines: *Matematika v šoli*, *Presek*. They are satisfied with the textbooks that are available.
Secondary school students use textbooks, worksheets, collections of exercises, the Internet; fourth-year students also use old matura examinations and textbooks for preparing for the matura examination.
Teachers say that they have the opportunity to undergo professional training and that the head teachers encourage this. All of the surveyed teachers, but one, are regularly attending such training programmes. They attend meetings of study groups, seminars, conferences, etc.
They find the offer of training programmes suitable and useful, whereas some say that the offer is insufficient. They wish for training programmes in working with disruptive students and with gifted students, while some wish for training programmes in the use of a specific computer programme: Geogebra, Excel (even though we have announced seminars on Dynamic Geometry Programmes and on Data Processing).

H3. Assessment of students is not aligned with the declared approach (concept) and teaching methods.

Not supported. In light of the traditional lessons, the assessment methods are appropriate; written tests are predominant. On written tests, teachers include shorter mathematical problem-solving tasks in a mathematical and real context.
They do not assess mathematical inquiries, except for statistical tasks, because they do not use them systematically in lessons.

RQ5: What (interesting) assessment methods do teachers use?

They most often carry out written assessment. They also carry out oral assessment, for instance, a product in statistics, a solved task and its presentation, a quiz (assessment via Moodle), a debate, product, competitions, etc.
They find written assessment to be the best method, because it provides the same conditions for all students and thus ensures objectivity. They have problems with oral assessment when there are too many students in the class; they do not have enough time for oral assessment.

RQ6: Are teachers interested in assessment method training?

Yes, and they are attending it. They speak highly of the seminar on drafting written examinations. They highlight that every training programme, whose knowledge they can directly transfer and apply to their own practice, is useful. They commend the exchanging of experiences with one another in the field of knowledge assessment.

H4. Teachers have a positive attitude towards changes in teaching and learning and have concreate ideas what and how to change on the macro (system) and micro (classroom) level.

Supported. The teachers believe that they have to constantly develop their own teaching practice and follow contemporary trends in the learning and teaching of Mathematics.

H5. Teachers feel that IBME brings some additional positive outcomes compared to the usual approaches and that it is worth the effort.

Supported. It can be indirectly established that teachers support contemporary learning and teaching methods (including IBME), but do not use them or do not know them well enough because of certain limitations.

RQ7: What ideas do teachers have for a change on a classroom level?

They believe that by reducing the number of secondary school students in a class and reducing the contents in the curriculum, we would achieve better results; however, not all share the opinion that an increase in the number of Mathematics lessons would improve achievements and knowledge.

The Netherlands

Prepared by: Rogier Bos, Utrecht University, The Netherlands

Teachers' background

Interviews were held with two teachers from the Stedelijk Gymnasium Utrecht, one from the Comenius College and one from the Hermann Wesselink College. The teachers were interviewed by Rogier Bos. During individual interviews only the interviewed teacher and the researcher were present. Two teachers were interviewed over Skype. All individual interviews were held in a very pleasant atmosphere and lasted more or less half an hour.

H1a. Teachers are aware of IBME approach and appreciate it but they do not implement it in classroom practice of mathematics teaching.

Mostly supported. Teachers have an imprecise idea of what IBMT is, but are positive about it. They do not consciously or structurally implement IBMT as we understand it.

One teacher has an idea of what it means, but has not found time to implement it in a systematic way. She started teaching mathematics again after a break of many years. She feels IBMT would be <u>not</u> of primary importance to implement. Another teacher gives a good example of IBMT, but says she does not precisely know what it is. A third teacher gives a sketchy description of IBMT. All teachers are positive about IBMT based on their own ideas. This is also implicitly clear from the fact that they want to participate in the project.

H1b. Teachers are aware of learning outcomes (LO) based approach and appreciate it but they do not implement it in classroom practice of mathematics teaching.

Not supported. Teachers did not have any clear idea of what the concept of LO means. Teachers are aware of the learning goals of their teaching and usually have a focus towards what will be tested. Teachers have not heard of the theory of Learning Outcomes. One teacher finds it important to have students formulate learning outcomes for every lesson. The teachers' approach is generally not based on theory or concepts. They think in terms of their goals (learning goals, but also "creating enthusiasm" or "making proper contact with the students") and how to achieve them using experience and practical knowledge. When asked, one teacher indicates that she **never** consciously bases a lesson on educational theory. Another teacher says she is not at all theoretical about her teaching and does not recall much theory of education. She states that she bases her teaching on practical experience.

RQ1: What other concepts they know and apply?

Two teachers mention they find it important to take a focus on higher cognitive skills, on mathematical thinking. This is not surprising, because there has been a lot of attention for this as part of the recent curriculum change in the Netherlands. Both also mention how important they find it to motivate their students. Another teacher mentions that the relation/communication with the students is a starting point for her teaching. She finds it important to connect to students' intuitions and wonder.

The interviewed teachers use classical instruction, classroom discussion, group work, instruction videos; sometimes games, competitions and quizzes. Sometimes groups of students work more independently of the teacher during the class, for example in the corridor of the school. The teachers try to bring variation to the classroom, but not consistently succeed. Teachers use a lot of group activities. For example: exercise merry-go-round. One teacher mentions the Socratic Dialogue method. He likes to use concrete contexts; in particular contexts he is familiar with from his previous work as a medical analyst.

Teachers (and students) use the book series, sometimes self-written materials, slide shows, Geogebra, Kahoot, Graphical calculator, Kahn Academy, Coursera, Youtube-clips, folding paper. Teacher B likes to sometimes bring concrete materials: like toilet rolls for cylinders, paper plates rotating for learning trigonometric functions, or pins and rope to draw conics.

H2a. Lack of adequate system support, teaching and learning material, teacher training and overloaded prescribed program are main reasons for not applying IBME approach in teaching on regular bases.

H2b. Lack of adequate teaching and learning material, teacher training and overloaded program are main reasons for not applying LO concepts in teaching on regular bases.

Partly supported. Teachers are just not so familiar with these teaching concepts. Therefore, there is no demand for such support or materials. Teachers do mention the overloaded program in this context.
There are many opportunities for teacher training in the Netherlands and teachers are aware of them. Some teachers say that inquiry-methods cost more time; this is difficult with the full curriculum. Also, inquiry methods require a lot of individual guidance which puts a high demand on the teacher. One teacher mentions the Dutch mathematics teacher's Facebook page, where many resources are shared.

RQ2: What other obstacles for not innovating teaching and learning process can be recognized?

One teacher says that she bases herself too much on the book, and also says using computers in classroom can be very difficult. There are always bugs and other obstacles. She mentions that her school joined in the Primas Project. She did not find it attractive to go to the website and search for materials. She says the level of the assignments did not seem to match what she was looking for.
Another teacher feels that teachers do not have enough time to prepare for teaching in new ways.

RQ3: What useful ideas teachers have that enable IBME?

This question was not part of the interview, so there was not enough data to answer it properly. In other sections of this reports the teaching ideas of the teachers are described. Some of them are related to IBMT.

RQ4: What kind of training and material teachers are interested in?

A teacher mentions she is interested in the courses and teacher training offered by the universities. Themes of these are, for example, subjects in the curriculum that are new (logic, analytic geometry) and mathematical thinking. Others have also taken some of these courses.
One teacher says that she would like to have help implementing IBMT in classroom. Another mentions explicitly that he would find this most important of all new things to learn and would like to learn to work with digital tools in his teaching. He feels he does not know enough about that.
They say that teacher trainings are often either to low level or they are unrealistic in what is achievable in classroom. All interviewed teacher like to go to the National Mathematics Days, a national conference for mathematics teachers.

H3. Assessment of students is not aligned with declared approach (concept) and teaching method.

Not enough data to decide on supported or not. There is little variation in the way students are assessed. The tests are hardly ever specifically aimed at higher order thinking skills, but neither is the teaching. They do try to have balanced traditional tests: not just reproduction items.

RQ5: What interesting assessment methods teachers use?

All teachers use summative paper tests. They base themselves on standard tests provided by the publisher of their text book. Some use formative tests in the forms of digital quizzes. One teacher once per year gives a test that is made in pairs. One teacher is interested in digital assessment; she is building experience with (formative) testing in the Dutch Digital Mathematics Environment.

RQ6: Are teachers interested in assessment method training?

Some teachers are very positive about learning new things and being inspired. Others are not particularly interested in this assessment training. They would like an assessment training together with her colleagues. Jointly they would like to develop ways of assessment that are less traditional. Some stress the importance to make these changes together. Some would like to be trained to implement high order thinking skills in their assessments.

H4. Teachers have a positive attitude towards changes in teaching and learning and have concrete ideas what and how to change on the macro (system) and micro (classroom) level.

Supported. Some teachers want a not so full curriculum. They want more time and freedom. On the other hand, one teacher wants a fuller curriculum (with more time). She also wants more time to experiment, in particular with mathematical thinking. Another teacher wants more mathematics hours for science students. This should not be optional, as it is now in the Netherlands (Mathematics D).

H5. Teachers feel that IBME brings some additional positive outcome compared to the usual approaches and that it is worth the effort.

Partly supported. Teachers are not well acquainted with IBME approach.

RQ7: What ideas teaches have for a change on a classroom level?

They would like to have their own classroom (not moving around all the time). One teacher wants to work on students' intrinsic motivation and would like to do bigger teaching experiments, stretching over more than one lesson.
They would like all students to have laptops: in particular to allow students to work in Digital Mathematics Environments. This in turn to allow differentiation. Some teachers would like the school to hire assisting staff for mathematics teaching, similar to the technical support staff for physics, chemistry and biology classes. They would like to reconsider the use of graphical calculator in classroom. One teacher is interested in curriculum comparison between countries and thinks the Dutch curriculum should be properly compared to other countries.

Croatia

Prepared by:
Blaženka Divjak, Faculty of Organization and Informatics, University of Zagreb; Aneta Copić and Eva Špalj, XV. Gymnasium, Zagreb; Matija Bašić and Željka Milin Šipuš, Faculty of Science, University of Zagreb, Croatia

Teachers' background

Interviews were held in the Business School Varaždin, Technical School Požega, Electro-engineering school Varaždin and XII Gymnasium Zagreb. In total, there were 13 teachers interviewed. Teachers were interviewed on the premises of their school by the above-mentioned researchers.
Teachers have read and signed the statement of consent at the beginning of the interview. During individual interviews only the interviewed teacher and the researcher were present. All individual interviews were held in a very pleasant atmosphere and lasted between 21 and 51 minutes (31 minute on average).
Interviewees teach mathematics in different programs which last 3 or 4 years. The interview focused on teaching and learning in four-year programs. Interviewed teacher's length of service varied from 2 to 26 years. All of them are fully qualified and they have graduated Mathematics Education at Faculty of Science, University of Zagreb.

H1a. Teachers are aware of IBME approach and appreciate it, but they do not implement it in classroom practice of mathematics teaching.

Partly supported. Teachers' appreciation of IBME varies, but their knowledge about it is mostly superficial. There is no implementation.
Only a few teachers have heard of the IBME approach, and they consider it useful. The others have not heard about IBME before, but after explanation, they confirmed that the approach seemed interesting and effective. With one exception, the teachers do not implement aforementioned approach, although they occasionally introduce some elements related to the problem- and inquiry-based learning. They provide following explanations for not using the IBME approach: students' attitudes (they expect routine tasks), students' relatively weak prior knowledge about the basic concepts of mathematics, students' lack of interest for mathematics, teachers' own lack of knowledge about IBME and the fact that such an approach requires more time in preparation and teaching that they do not have.

H1b. Teachers are aware of learning outcomes (LO) based approach and appreciate it but they do not implement it in classroom practice of mathematics teaching.

Partly supported. Teachers are aware of the LO, while their appreciation varies; usage is claimed, but it is hard to conclude to which level the constructive alignment is achieved because of evident misinterpretations.
All of the teachers recognize learning outcome concept because it is a part of the mandatory curriculum (national legislation), and they are able to formulate learning outcomes. On the other hand, further conversation shows that teachers

often speak about learning outcomes that they (teachers) should accomplish, and not the students.
Teachers' attitude towards the learning outcomes vary. Some teachers are positive about it and emphasized that it would be difficult to organize teaching without defining learning outcomes. Those who have negative attitudes claim it is a novel concept and that they are not yet used to it. They also associate additional administration with the use of learning outcomes.
Teachers claim to implement the concept of learning outcomes in preparation, reflection and design of the exams. Students generally are not informed about the intended learning outcomes. It cannot be concluded from the answers that teachers align learning outcomes with the methods of teaching and assessment. Teachers point out that they would like to have more specific training about the use of learning outcomes in mathematics and how to achieve constructive alignment of learning outcomes, teaching methods, and assessment methods.

RQ1: What other concepts teachers know and apply?

They commonly use a frontal (ex cathedra) approach in teaching mathematics, but all of them often introduce elements of problem based and inquiry based approach, dialogue method, and heuristic approach. Games and quizzes are often mentioned as means used to rise motivation and activation. The concept of flipped classroom is a novel concept that some teachers find interesting and effective. Teachers find that it is important that students take an active role and that this kind of activities increase students' motivation.
Revision tasks are done in pairs or groups (collaborative work). In general, teachers check homework, but report that students in higher grades quit writing their homework. They ask a student to explain the solving process to avoid cheating and copying.
While teaching new lessons teachers try to start with an interesting and motivating practical example.
Example:

> Teacher started right triangle trigonometry with two examples: the slope of the ramp for the disabled – is the ramp built according to the regulations (example from the book) and limitation of the mower on the 15% slope (teacher's example).

Only a few teachers elaborate on the development of the lesson. One interesting description:

> *During the discussion (guided conversation) students become aware that the task cannot be solved, therefore the lesson continues with simpler tasks (solved by the students individually or on the board), introducing some new concepts that help solving the motivational problem. At the end, the knowledge is systemized and the homework is assigned.*

Teachers and students use tools like Geogebra, computers, materials from the Internet, and the anecdotes from the history of mathematics. In one school, there is also an experimental program "Mathematics and Informatics" which applies interesting approaches to learning.

They promote the approach that mathematics is not a “bogeyman" and they think it is important to get students accustomed to different methods of teaching and to insist on the development of logical reasoning.

H2a. Lack of adequate system support, teaching and learning material, teacher training and overloaded prescribed program are main reasons for not applying IBME approach in teaching on regular bases.

Supported. As systematic challenges it is pointed that students do not have the required knowledge or the positive attitudes towards mathematics after primary school. There is a lack of time to prepare own materials, specialized training about contemporary methods are missed and overloaded prescribed program is emphasized as a big problem.

Teachers in different schools emphasize different problems. According to some, the biggest problem for applying IBME are students’ negative attitude towards mathematics formed in primary school and extremely weak prior knowledge at the end of primary school.

Furthermore, there aren’t enough specialized trainings for teachers about contemporary methods of teaching. Also, there are no textbooks and quality work materials which would support inquiry based and other contemporary teaching methods. Although some schools are relatively well equipped and grants access to other teaching methods due to e-learning, most schools should be equipped better.

Other teachers point out that the programme is too extensive for the prescribed number of teaching hours and that they would like to have more time to process the material. For them, the biggest problem is lack of time due to overloaded program and their own lack of knowledge how to implement IBME.

In all schools, collaboration among teachers is excellent, but there is almost no cooperation outside of school. There are seminars on the mathematical teaching topics but there isn’t always a full benefit from the exchange of ideas. Teachers find seminars accessible, but sometimes the quality does not justify the length. The most important training would be to present concrete example of good practice and transfer the know-how on design of new materials (or obtaining ready-to-use materials).

H2b. Lack of adequate teaching and learning material, teacher training and overloaded program are main reasons for not applying LO concepts in teaching on regular bases.

Partly supported. Teachers use the concept of learning outcomes as it is part of the mandatory curriculum, but the benefits are not fully achieved due to lack of understanding the concept, omission of the concept in textbooks and negative attitude rising from additional administration associated with the concept.

In general, teachers clearly claim that they use the learning outcomes in preparation, implementation and assessment phase. This is not supported by examples. In addition, most teachers are satisfied with textbooks and only miss the link to everyday life. Some teachers associate additional administration with the use of learning outcomes, and they do not recognize benefits.

Teachers are designing learning outcomes, but they think that their students would not be interested in it. All teachers have undergone a training on learning outcomes, but they think it would be important to explain to them what a learning outcome in mathematics is. It's challenging for teachers to plan their classes based on learning outcomes because LOs were not anticipated in the textbooks. In school, teachers often communicate, they use the Internet to prepare classes, and they consider that the use of technology has inspired students to better achieve learning outcomes. They consider that the problem are textbooks and tests (along with the training tests) in the primary school because that's how students get used to the narrow way of working, so they resist learning that is related to logical reasoning, self-regulation and mathematical modelling. Teachers, apart from those who have recently graduated, did not attend quality training on the use of learning outcomes in teaching, especially in the evaluation of the students' work. They ask for more concrete examples.

RQ2: What other obstacles for not innovating teaching and learning process can be recognized?

Teachers feel that changes in primary education are necessary so that students would first learn basic mathematical concepts because secondary school curriculum are too extensive to repeat the basics. Targeted trainings for teachers can also help. Teachers cannot get support in some specific topics and initiatives. More additional teaching materials are necessary so that the teachers wouldn't have to make them themselves.

Students are immersed in routine tasks and their expectations pose a problem. Some teachers report that students follow basic general instructions (e.g. find a part of text in the book or to design a poster) with difficulties as they cannot discern what is important. As a consequence, students are not motivated, and despite the ability they show in class, they do not have good results at the end of cycle exams. It is a challenge for teachers to organize group work so that all group members are active. Logical thinking is set in the background, but one should certainly insist on it. Some schools are poorly equipped.

RQ3: What useful ideas teachers have that enable IBME?

The school has a combined mathematics and informatics experimental course. In this combined course students receive project tasks (e.g. calculating absence statistics in the classroom). Students and the teacher get a common grade and considering this is an experimental course, the assessment is taken more lightly. Methods such as group work or work in pairs are also used but such learning is difficult to assess.

Teachers agree that teaching must not be monotonous, they emphasize the need to alternate teaching methods, use the computer, etc. Sometimes they are looking for critical reflection about teachers' claims to encourage self-confidence in students. They find that it is important that students take an active role and that this kind of activities increase students' motivation. In general, teacher's main aim is to build a relationship with the students in order to establish a working and inspiring atmosphere and to engage students.

RQ4: What kind of training and material teachers are interested in?

Teachers are interested in targeted training related to mathematics: learning outcomes in mathematics and assessment in mathematics, and especially the evaluation of projects (rubric). They are also interested in new teaching methods and approaches such as inquiry based learning (IBME).

They think it would be beneficial to have extra materials in the form of e-learning. Training on e-learning and of assessment of complex tasks, group work and homework, is also needed. They would like to have more material for students' work in pairs or in groups. In general, they need more material that will supplement the textbook in terms of problem and inquiry based approaches.

They consider that it is crucial that training for teachers generates concrete materials and examples (not necessarily detailed) for specific topics and to abandon frontal (ex cathedra) work with the teachers.

Some teachers report that they are unqualified to deal with gifted students and students with disabilities, and they think this should change with training.

H3. Assessment of students is not aligned with declared approach (concept) and teaching method.

Supported. Teachers find that assessment is the hardest part of their job and they are not confident about assessing various students' activities (anything different from written exams). They must evaluate student activities they find irrelevant because the exams do not reflect the abilities noticed during lessons.

Generally, assessment is not systematically aligned with the teaching methods and learning outcomes. There is also an intention "from above" (the level of the national Agency) to define the elements of evaluation. Some teachers claim that they must evaluate student activities they find irrelevant because the exams do not reflect the abilities that teachers notice during lessons. Young teachers find that assessment is a challenge as they feel they are not assessing all activities students are engaged in and they miss guidelines. It seems that assessment methods of teachers are a domain where additional training is greatly missed. Teachers claim that they think about learning outcome while designing exam questions, but they have not offered clear examples supporting that. Teachers mostly do not have problems with assessment of tests and exams, but they find problem tasks, group work, homework (cheating), and sometimes even the oral examination challenging to assess. Young teachers miss clear guidelines.

RQ5: What interesting assessment methods teachers use?

They mostly use standard methods such as tests, oral examination and homework. Oral exams are rarely used as they feel that other students are neglected during that time. They do not use explicit evaluation criteria or a rubric. Most teachers assess commitment (activity in the classroom) using own system of pluses and minuses. Formative assessment is established within the discussion about common mistakes, or by informing students about their progress during exercise class.

Sometimes they assign projects as homework (e.g. designing a poster). Some teachers use rewarding tasks and incentive rating for e.g. students' posters or presentation. However, they don't evaluate the latter the same way they evaluate

tests. In the experimental lesson (mathematics and Informatics) they also use project tasks with the help of a computer, but they find it hard to evaluate. They sometimes introduce working in groups, but that method is difficult to assess.

RQ6: Are teachers interested in assessment method training?

They are interested, especially for the evaluation of complex tasks and peer assessment, which is implicitly specified because some problems related to peer evaluation were cited as well as how to use assessment rubric for complex task assessment (problem or project work).

Teachers who have recently graduated had the course Assessment for which they claim to have been useful to their teaching practice. Therefore, they know more about the topic of assessment but still rarely use some non-standard methods of assessment.

In general, teachers find that assessment is the most challenging part of their job. More experienced teachers have developed their own methods, while younger lack guidance. They generally consider assessment to be the hardest part of the teaching, but they also think that assessment can motivate students to learn mathematics. At the end of June there will be national conference on evaluation and assessment. But they still want to learn more about it.

H4. Teachers have a positive attitude towards changes in teaching and learning and have concrete ideas what and how to change on the macro (system) and micro (classroom) level.

Supported. Interviewed teachers are enthusiasts and they look forward to improvements. They are very open to learning, applying ICT in teaching and learning, and would like changes towards reducing the pressure on students in terms of prescribed program and number of subjects.

Even though teachers claim to have many students with weaker grades in mathematics who dislike mathematics, teachers have a positive attitude about how getting extra education and making changes in teaching and learning can lead to achieving better results.

Some teachers have difficulties in naming concrete changes in teaching methods, but they think that the number of subjects is too big, there is not enough hours per week for mathematics and that students are facing significant pressure.

They consider that the students do not have the initiative (self-regulation) to independently explore and learn but that they can and should be taught to do it. A teacher is there to motivate and to encourage students to learn. Teachers use interesting approaches that have already been described earlier. Teachers are open to change their practice. For them, the development of logical reasoning and critical thinking, and linking mathematics with practice at vocational schools are important.

H5. Teachers feel that IBME brings some additional positive outcome compared to the usual approaches and that it is worth the effort.

Partly supported. Teachers are not well acquainted with IBME approach. They assume that there are benefits of the approach, but they mostly recognize challenges: students' low motivation and weak knowledge.

IBME approach is not explicitly recognized, and teachers think that it is particularly difficult to implement such an approach when students have weak knowledge and poor motivation.
Learning critical and logical reasoning is hard for students in high school. Teachers mostly think that there are students with whom they can work, but there are not a lot of them. Teachers are trying to encourage students to ask questions and react if they have a different opinion. However, such approach is especially difficult in the first grade because students are not trained for it.
Some think that IBME is valuable as an approach because the knowledge acquired in that way has better retention, while others consider that this approach can work if and only if students are motivated. It is important that students learn from concrete situations to see what the purpose of the knowledge being taught is.

RQ7: What ideas teachers have for a change on a classroom level?

Some teachers prefer an interdisciplinary approach (mathematics and informatics), the use of computers (e-learning), and the others prefer examples from real life (e.g. loans) that are related to students' future profession. But some teachers stress that not all real-life examples are relevant and motivating. They point out that, if they had more time, they would like to implement group work and work in pairs, and alternative approaches such as building posters, presentations, models for teaching, etc. They would like to spend more time teaching through concrete situations that would enable them to show applications of mathematics and provide better understanding (what is this good for and why do we need this).
To implement those approaches, teachers need additional education. Textbooks are not following modern teaching and learning concepts, teachers are preparing their own materials or use material for group work which they found in professional journals. They think that with more support contemporary approaches would be more widespread.
Most teachers conclude that they would like to reduce the number of subjects in secondary school and increase the number of hours per week for mathematics.

Other comments

Finally, teachers had a lot of comments on the national education system. They think that students in elementary school should not be prepared for the exams with practice exams that resemble real exams. It is necessary to reduce students' obligations in elementary school so that they could master basic concepts. All Croatian schools should be equally well equipped (e.g. tools and computers). It is necessary to organize more trainings for teachers and to enable all of them to participate and not just a few teachers (funding). It is particularly important to connect mathematics to the real life, but in a relevant way and to promote a way of thinking that school is not just an obligation but a preparation for real life. It is important to somehow ensure a smooth transition from the elementary school to secondary school. Teachers believe that self-regulated learning and development of independence in learning should be encouraged. For now, higher metacognitive skills could not be developed fully in their school because of the lack of time. It certainly would be useful to improve materials that would ease the use of it.

Joint report for all four countries

Prepared by:
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H1a. Teachers are aware of IBMT approach and appreciate it, but they do not implement it in classroom practice of mathematics teaching.

H1a is partly supported in all countries. Teachers' appreciation of IBMT varies in Croatia and Slovenia, but their knowledge about it is mostly superficial. In the Netherlands and Denmark all teachers, in spite of their imprecise understanding of IBMT, support this way of teaching and learning. There is no clear indication of implementation of IBMT, although in all countries teachers occasionally introduce some elements related to the problem-based and inquiry-based learning.

H2a. Lack of adequate system support, teaching and learning material, teacher training and overloaded prescribed program are main reasons for not applying IBMT approach in teaching on regular bases.

H2a is supported in Slovenia and Croatia, while only partly supported by the data in the Netherlands and Denmark. As systematic challenges in Slovenia and Croatia it is pointed that students do not have the required knowledge or the positive attitudes towards mathematics after primary school. Teachers lack time to prepare own materials, and specialized training about contemporary methods is missed. Overloaded prescribed program is emphasized as a big problem in Slovenia, Croatia and the Netherlands as well. In Denmark a big curriculum reform was ongoing at the time of the interview, so teachers could not judge if the curriculum will be overloaded or not.

Teachers in different schools emphasize different problems. According to some, the biggest obstacles to apply IBMT are students' negative attitude towards mathematics formed in primary school and relatively weak prior knowledge at the end of primary school. They also mention students' lack of interest for mathematics and teachers' own lack of knowledge about IBMT. Furthermore, there are not enough specialized trainings for teachers about contemporary methods of teaching. Also, there are no textbooks and quality work materials which would support inquiry-based teaching and other contemporary teaching methods. Although some schools are relatively well equipped and grant access to other teaching methods due to e-learning, most schools should be equipped better. Other teachers point out that the programme is too extensive for the prescribed number of teaching hours and that they would like to have more time to process the material. For them, the biggest problem is lack of time due to overloaded program and their own lack of knowledge how to implement IBMT. In all schools, collaboration among teachers is excellent, but there is less cooperation outside of school.

Hypothesis H2a consists of four variables. In the following table, we have summarized the findings for each variable separately.

	Lack of adequate system support	Lack of teaching/learning materials	Lack of teacher training	Overloaded prescribed program	Supporting H2a
CROATIA	YES -negative attitude -no required knowledge	YES -lack of time to prepare	YES -about contemporary methods	YES -big problem	Supported
SLOVENIA	YES -aware of advantages, but rarely implement it	YES	YES	YES -students are overburdened	Supported
THE NETHERLANDS	NO -because they are not so familiar with these teaching concepts		NO -they have many opportunities	YES -overload reported in general	Partly supported
DENMARK	YES -school pays an additional course, but don't give extra time for it	NO	NO -nobody has to use special material for teaching -opportunity to attend course	N/A CURRICULUM REFORM	Partly supported

Table 1. Overview of results to H2a.

H1b. Teachers are aware of learning outcomes (LO) based approach and appreciate it but they do not implement it in classroom practice of mathematics teaching.

H1b is partly supported in all countries. The term *learning outcome* is not in widespread use in all countries, although similar terms are used for the similar (or the same) concept. The concept is also understood under the terms expected achievement, learning goal or target knowledge. It is hard to decide how to interpret the answers and to conclude to which level the constructive alignment is achieved because of evident misinterpretations.

Data from Croatia might be most enlightening. In Croatia, all of the teachers recognize learning outcome concept because it is a part of the mandatory curriculum (national legislation), and they are able to formulate learning outcomes. On the other hand, further conversation shows that teachers often speak about learning outcomes that they (teachers) should accomplish, and not the students. Teachers' attitude towards the learning outcomes vary. Some teachers are positive about it and emphasized that it would be difficult to organize

teaching without defining learning outcomes. Those who have negative attitudes claim it is a novel concept and that they are not yet used to it. They also associate *additional administration* with the use of learning outcomes. Teachers claim to implement the concept of learning outcomes in preparation, reflection and design of the exams, but the students generally are not informed about the intended learning outcomes.

H2b. Lack of adequate teaching and learning material, teacher training and overloaded program are main reasons for not applying LO concepts in teaching on regular bases.

As for H1b, it was hard to make conclusions because of different interpretations and misinterpretations of the concept of learning outcome in different countries. Partly supported only in Croatia. Teachers use the concept of learning outcomes as it is part of the mandatory curriculum, but the benefits are not fully achieved due to lack of understanding the concept, omission of the concept in textbooks and negative attitude rising from additional administration associated with the concept.

In the Netherlands and Denmark concept was not recognized explicitly. In Slovenia teachers are familiar with the concept of expected achievements and they use it.

	Lack of teaching/learning materials	Lack of teacher training	Overloaded prescribed program	Supporting H2b
CROATIA	YES -omission of the concept in the textbook	YES -lack of understanding of the concept	YES -negative attitude rising from additional administration	Supported
SLOVENIA	NO - teachers use expected achievements without obstacles	NO	NO	Not supported
THE NETHERLANDS	N/A - no recognition of the concept	N/A	N/A -overload reported in general	No conclusion
DENMARK	N/A - no recognition of the concept	N/A	N/A CURRICULUM REFORM	No conclusion

Table 2. Overview of results to H2b.

RQ1: What other concepts teachers know and apply?

Some answers are common in all countries and mentioned by a majority of teachers as they all use frontal lecturing, dialogue method, heuristic approach and some form of collaborative work. Teachers and students use ICT in all countries. Games and quizzes are often mentioned as means used to rise motivation and activation. Besides motivation, Dutch teachers find it very important to encourage development of higher cognitive skills. While teaching new lessons teachers try to start with interesting and motivating concrete example. All mentioned ideas are summarized in the following table. It is apparent from the list that the teachers do not think in terms of a particular teaching approach or educational theory.

Concepts in mathematics education mentioned by the teachers		
frontal, classical instruction	students take an active role	higher cognitive skills
inquiry-based approach	learning to learn	motivation
guided conversation	modelling	relation/communication with the students
Socratic dialogue method (use concrete contexts), classroom discussion	independent learning	use of ICT: graphical calculator, Geogebra, slide shows, Kahoot, Kahn Academy, Coursera, instruction videos, YouTube-clips
problem-based approach and problem solving	homework	anecdotes from the history of mathematics
reversed classroom	cross-curricular integration	development of logical reasoning
heuristic approach	use of concrete examples to understand concepts	folding paper, concrete materials (like paper plates)
collaborative work, working in pairs	interesting and motivating practical examples	text-book (series), self-written materials
games and quizzes, competitions		

Table 3. Concepts in mathematics education mentioned by the teachers.

RQ2: What other obstacles for not innovating teaching and learning process can be recognized?

Obstacles mentioned by the teachers are similar in different countries, although the focus is slightly different. Slovenian and Croatian teachers feel that changes in primary education are necessary so that students would first learn basic mathematical concepts because secondary school curriculum is too extensive to repeat the basics. Slovenian teachers regret that schools are too efficacy oriented, especially because of the matura exam. Targeted trainings for teachers can also

help. Teachers cannot get support in some specific topics and initiatives. More additional teaching materials are necessary so that the teachers would not have to make them themselves and could save time which is already a problem for the most of them.

Students are immersed in routine tasks and their expectations pose a problem. As a consequence, students are not motivated and, despite the ability they show in class, they do not have accordingly good results at the end of cycle. Logical thinking is set in the background, but one should certainly insist on it. Some schools are poorly equipped in Croatia, while Dutch teachers complain on constant technical problems with computers. Data gathered in Denmark was not sufficient for analysis.

Croatia	Slovenia	Netherlands
students' low motivation	education system needs to change	base themselves too much on the book
students' weak knowledge from primary school and necessity to revise	the school is too efficacy-oriented	using computers in classroom can be very difficult (there are always bugs and other obstacles)
lack of time to prepare materials	the teacher is too limited by the curriculum and pressured by the matura examination	the level of the exercises found on the web did not seem to match what was needed
lack of time for student activities due to overloaded program	lack of time for inquiry-based tasks	lack of time to prepare for teaching in new ways
lack of know-how to organize group activities and engage all students	students overestimate themselves and then limits have to be lowered	
setting logical thinking in the background	faculty does not provide sufficient knowledge and they are not in touch with the actual conditions in school	
poor technological equipment of schools		

Table 4. Obstacles to implementing innovative teaching and learning processes

RQ3: What useful ideas teachers have that enable IBMT?

Croatian teachers mention the following ideas that are already implemented in their work: a combined mathematics and informatics experimental course, the need to alternate teaching methods, use of ICT, critical reflection about teachers' claims to encourage self-confidence in students, building a relationship with the students in order to establish a working and inspiring atmosphere and to engage students.

In Slovenia, teachers' responses include: use of ICT, providing more meaning to mathematical contents, increasing motivation, improving pedagogical skills for dealing with students with special needs, reducing the role of assessment.

More ideas were mentioned for RQ1 and presented in Table 3.

RQ4: What kind of training and material teachers are interested in?

There are seminars (Croatia) and courses (Denmark) on topics related to teaching mathematics, but there is not always a full benefit from the exchange of ideas. Teachers find these activities accessible, but sometimes the quality does not justify the length. Responses are similar in all countries. Croatian teachers point out their interest in training on assessment, Dutch on innovative teaching methods (as IBMT), while Slovenian teachers mention e-learning and using ICT in general. Dutch teachers mention the National Mathematics Days, a national conference for mathematics teachers, as an example of a respectful event. In general, teachers need more material that will supplement the textbook in terms of problem- and inquiry-based approaches. They consider that it is crucial that training for teachers generates concrete materials and examples (not necessarily detailed) for specific topics. They would also like to have more materials that can be used for pairs and groups.

H3. Assessment of students is not aligned with declared approach (concept) and teaching method.

It is not easy to make a clear statement if hypothesis H3 is supported in any country due to the complexity of approaches used and teachers' understanding of the alignment.

In Croatia, teachers find that assessment is the hardest part of their job and they are not confident about assessing various students' activities (anything different from written exams). They must evaluate student activities they find irrelevant because the exams do not reflect the abilities noticed during lessons. In general, assessment is not systematically aligned with the teaching methods and learning outcomes. There is also an intention "from above" (the level of the national Agency) to define the elements of evaluation. Young teachers miss clear guidelines.

From the data in Slovenia, one may conclude that teachers use a traditional style of teaching and traditional methods of assessment, along with the use of the concept of expected achievement. On the other hand, answers to other questions show that there are other non-traditional approaches used occasionally and there is no reflection on non-traditional methods of assessment that connects teaching to assessment.

In the Netherlands, there is little variation in the way students are assessed. The tests are hardly ever specifically aimed at higher order thinking skills, but neither is the teaching. They do try to have balanced traditional tests: not just reproduction items.
In Denmark, there are various assessment methods used at the end of the course – classical written exams, projects with written reports and oral exams in a form of a dialogue. Assessment is oriented towards these final exams, but as these are diverse it supports different skills.
Although data is not sufficient to make clear conclusions, this very fact shows that teachers are not completely aware of the concept of constructive alignment and hence do not use it consciously.

RQ5: What interesting assessment methods teachers use?
All teachers mostly use standard methods such as tests, oral examination and homework. They do not use explicit evaluation criteria or rubrics. In the Netherlands and Slovenia some digital assessment tools are used, such as quiz made in Moodle (learning management system). In Croatia, in the experimental course (mathematics and informatics), digital project tasks are used, but teachers find it hard to evaluate students' work. Most teachers in Croatia assess commitment (activity in the classroom) using their own system. Formative assessment is established within the discussion about common mistakes or by informing students about their progress during exercise class.
Slovenian and Croatian teachers sometimes assign projects as homework (e.g. designing a poster or preparing a presentation). However, they do not evaluate these the same way they evaluate tests. They sometimes introduce group work or open discussions, but these forms are difficult to assess.
In general, teachers find that assessment is the most challenging part of their job. More experienced teachers have developed their own methods, while younger lack guidance. They also think that assessment can motivate students to learn mathematics.

Assessment methods		
summative assessment: standard written and oral examination	homework	digital quizzes, assessment using Moodle
formative assessment: discussing common mistakes and providing feedback during exercise class	competitions	assessment of commitment (activity in the classroom)
rewarding tasks and incentive rating for e.g. students' posters or presentation	exam solved in pairs	

Table 5. Use of assessment methods

RQ6: Are teachers interested in assessment method training?

Teachers are interested in assessment method training, especially on the evaluation of less traditional methods and complex tasks, peer assessment and digital assessment in general.

In Croatia, teachers who have recently graduated had the course *Assessment in mathematics education* for which they claim to have been useful to their teaching practice. Their answers on the on the other hand show that they are rarely confident to use non-standard methods of assessment. Teachers point out that they would like to have *more specific training* how to achieve constructive alignment of learning outcomes, teaching methods, and assessment methods. It would be important to present concrete examples of good practice and transfer the know-how on design of assessment methods. In Slovenia, teachers highlight that every training programme, whose knowledge they can directly transfer and apply to their own practice, is useful. They commend the exchanging of experiences with one another in the field of knowledge assessment.

Croatia	Slovenia	The Netherlands	Denmark
Yes, in particular for the evaluation of complex tasks and peer assessment	Yes, and they are attending it. They speak highly of the seminar on drafting written examinations.	Mostly yes, for example for less traditional assessment methods and evaluating high order thinking skills.	Mostly yes, but the time for professional development is not part of the contract.

Table 6. Interest for assessment method training.

H4. Teachers have a positive attitude towards changes in teaching and learning and have concrete ideas what and how to change on the macro (system) and micro (classroom) level.

Supported in all countries. Interviewed teachers are enthusiasts, look forward to improvements and have a great willingness. They are very open to learning, applying ICT in teaching and learning, and would like changes towards reducing the pressure on students and teachers in terms of prescribed program and number of subjects. Most of them are constantly improving their practice.

Even though teachers claim to have many students with weaker grades and lower interest in mathematics, teachers have a positive attitude about their own additional education and making changes in teaching and learning that can lead to better results. They consider that the students lack initiative (self-regulation) to independently explore and learn, but that they have the abilities and should be taught to do it. They recognize that the role of teacher is to motivate and to encourage students to learn. Teachers use interesting approaches that have already been described earlier.

	Positive attitude towards changes	Have ideas what and how to change	Supporting H4
CROATIA	YES - enthusiasts - very open to learn	YES - reducing the pressure on students	Supported
SLOVENIA	YES - follow contemporary trends	YES - more time	Supported
NETHERLANDS	YES - follow contemporary trends	YES - more time and freedom to experiment	Supported
DENMARK	YES - great willingness	YES - most of them are continuously improving their practice	Supported

Table 7. Attitudes towards change

H5. Teachers feel that IBMT brings some additional positive outcome compared to the usual approaches and that it is worth the effort.

Partly supported in all countries. Teachers are not well acquainted with IBMT approach. They assume that there are benefits of the approach, but they mostly recognize challenges: students' low motivation and weak knowledge. Logical reasoning seems hard for students in high school, as well as developing high order cognitive skills. Teachers sometimes think in terms of students with whom they can work and whom they cannot. Nonetheless, teachers are trying to encourage students to ask questions and react if they have a different opinion. However, in Croatia teachers say such approach is especially difficult in the first year of high school because students are not used to it and the reactions are absent.

	IBMT brings some additional positive outcome	IBMT is worth the effort	Supporting H5
CROATIA	YES	NO	Partly supported
SLOVENIA	YES	N/A - do not know enough about it	Partly supported
NETHERLANDS	YES	NO - not well acquainted	Partly supported
DENMARK	YES - great willingness	N/A	Partly supported

Table 8. Appreciation of IBMT

RQ7: What ideas teachers have for a change on a classroom level?

In the Netherlands, teachers would like to have more freedom (more time, less full curriculum, their own classroom). They think about how to improve students' intrinsic motivation and use ICT to allow differentiation. They also suggest hiring assisting staff for mathematics teaching, similar to the technical support staff for physics, chemistry and biology classes. Interest in comparison of Dutch curriculum to other countries is mentioned, too.

In Denmark, there was a new reform with the first students starting in August 2017. There are many changes especially in terms of content of the curriculum for mathematics, but also in the didactics. Teachers were waiting and getting ready to see the changes.

In Slovenia, teachers believe that by reducing the number of secondary school students in a class and reducing the contents in the curriculum, they would achieve better results; however, not all share the opinion that an increase in the number of Mathematics lessons would improve achievements and knowledge.

In Croatia, some teachers prefer an interdisciplinary approach (mathematics and informatics), the use of computers (e-learning), and the others prefer examples from real life (e.g. loans) that are related to students' future profession. Some teachers stress that not all real-life examples are relevant and motivating. They point out that if they had more time they would like to implement group work and work in pairs, and alternative approaches such as building posters, presentations, models for teaching, etc. They would like to spend more time teaching through concrete situations that would enable them to show applications of mathematics and provide better understanding (what is this good for and why do we need this). To implement those approaches, teachers need additional education. Textbooks are not following modern teaching and learning concepts, teachers are preparing their own materials or use material for group work which they found in professional journals. They think that with more support contemporary approaches would be more widespread. Most teachers conclude that they would like to reduce the number of subjects in secondary school and increase the number of hours per week for mathematics.

Finally, teachers had a lot of comments on the national education system. They think that students in elementary school should not be prepared for the exams with practice exams that resemble real exams. It is necessary to reduce students' obligations in elementary school so that they could master basic concepts. All Croatian schools should be equally well equipped (e.g. tools and computers). It is necessary to organize more trainings for teachers and to enable all of them to participate and not just a few teachers (funding). It is important to somehow ensure a smooth transition from the elementary school to secondary school. Teachers believe that self-regulated learning and development of independence in learning should be encouraged. For now, higher metacognitive skills could not be developed fully in their school because of the lack of time. It certainly would be useful to improve materials that would ease the use of it.

Conclusions influencing the project team

Prepared by: Matija Bašić, Faculty of Science, University of Zagreb, Croatia

Through these interviews valuable qualitative data that gives a deep insight into the teaching of mathematics, the challenges faced by teachers and their needs has been collected. In addition, teachers gave a whole range of useful examples of good practice. Finally, there are valuable information for project team to streamline further project activities.

Teachers' understanding of IBMT and Attitude towards change

Based on the findings related to H1a, H2a, H4 and H5 we conclude that the interviewed teachers share an interest towards inquiry-based approach and a positive attitude towards making changes in their teaching. Concerning concepts, their familiarity with IBMT is not very deep and they would welcome instructions on implementation of alternative scenarios. Based on answers to RQ1, we conclude that teachers do not think in terms of a specific educational theory and do not implement IBMT, although in all countries teachers occasionally introduce some elements related to the problem-based and inquiry-based learning. We have listed many ideas gathered from the teachers in Table 3.

Teachers assume that there are benefits of the IBMT approach, but they mostly recognize challenges. Based on answers to RQ2, the main obstacles to applying IBMT that teachers report are lack of time to prepare (innovative) lectures, students' motivation for learning mathematics, lack of quality materials supporting IBMT and general orientation on final exams (assessment) and routine tasks. In addition, teachers are sceptical about the classroom and time management, so they would be more confident to use new materials only if they are well prepared. We may say that the interviewed teachers are very enthusiastic. This is not a surprise since they agreed to participate in the project, but we should be aware that they are above the level of motivation shown by the average teachers.

For the project this means that in teachers we have interested partners, who seek additional education in IBMT. We also note that for teachers to accept a new approach, we need to convince them that it is worth their (in-classroom and preparation) time. With this perspective, we have designed *MERIA Practical Guide to Inquiry-Based Mathematics Teaching*, as well as *MERIA Scenarios and Modules* (see section Materials below), which will be disseminated through *MERIA workshops* (see section Training below).

Concerning systematic changes, answers to RQ7 show that teachers in different countries have somewhat different problems and concerns. In Denmark many questions were hard to answer at the time because of the curricular reform. In the Netherlands, teachers emphasize they would like more freedom in organization of teaching. In Croatia on the other hand, teachers seek modernization of teaching approaches and support in using contemporary methods. They also find that the curriculum and the teaching methods are not well aligned between elementary and high school. In Slovenia, teachers would like to reduce the number of students in a class and the number of subjects.

Understanding of learning outcomes

Concerning learning outcomes (H1b and H2b), it was hard to gather, analyse and compare results in different countries due to different use of terminology. Similar concepts of learning goals and target knowledge are also widely used. It is apparent from the interviews that teachers do think about what their students need and will know at the end of the cycle, but it is not clear to which extent the constructive alignment is achieved (in preparation, teaching and assessment). MERIA Scenarios and Modules will use the notion of Target knowledge and Broader goals in the scenario, while the rationale behind the scenario will be explained in the module. Module will also contain short exercises designed to test if the target knowledge has been achieved. Finally, these issues will be addressed during workshops (and hence described in the MERIA Workshop Guide).

Ordinary math class and Learning and teaching materials

Teachers use a variety of teaching methods and they are very much willing to try out new ideas once they are convinced that the novelty is worth their time. For the project it is thus crucial to develop *MERIA scenarios and modules* at high quality followed by convincing dissemination.

Most of the teachers have the feeling, that students like the same structure in the math lessons and are motivated working that way. A structure often used is

- Introduction to the lesson
- Repetition of concepts from last time
- Introduction to new concepts and methods
- Students do exercises using the new concepts and methods
- Concluding the lesson

Teachers also describe the role of certain parts of the structure in their own words:

> *During the discussion (guided conversation) students become aware that the task cannot be solved, therefore the lesson continues with simpler tasks (solved by the students individually or on the board), introducing some new concepts that help solving the motivational problem. At the end, the knowledge is systemized, and the homework is assigned.*

To support their teaching, they mostly use textbooks, but quite often they look for inspiration online or though professional development. In addition to structure, teachers show that they are aware of the importance of motivation and relevancy for learning.

Within project MERIA, the structure of the lessons has been adopted from the Theory of Didactical Situations which has been disseminated to teachers through MERIA workshops. Ideas for MERIA scenarios should be selected to meet the criteria of relevancy and applicability. MERIA Modules come equipped with explanation of the rationale behind the scenario, additional materials and experiences from testing the scenario. This whole development in the project is carried based on the input gathered from teachers included in these interviews.

Assessment

The issue of assessment in general has been addressed with H3 and RQ3. Interpretation of the results is delicate. We conclude that the alignment of assessment and the teaching approach is achieved only when teachers use traditional methods (frontal approach combined with classroom discussions, a lot of practice and written exams). Assessment in general is one of the greatest challenges for the teachers and they often mention that they would appreciate further training.

The aim of the project MERIA is to promote IBMT, knowing that it requires long time for students and teachers to adapt to the change, so emphasizing that in the beginning we must encourage interest, engagement and free interaction in the classrooms and there should be no pressure in terms of assessment as that would ruin the potential for learning. In that respect, we have set a goal for the project to provide more ideas for formative assessment and achievement of constructive alignment throughout MERIA workshops.

Training

The supply of teacher training is abundant in all countries, but the quality of the offer varies. Teachers find some workshops very useful, while some are, in their opinion, a waste of time. Only singular topics or events are mentioned and there is no mention of any form of systematic continuous professional development. Our interpretation is that longer overarching programmes of teacher training are missing. Meetings with teachers in associated schools were hence more frequent than it was planned in the project application. MERIA workshops are not organized as disconnected events, but as a series of workshops with opportunities for teachers to try out and reflect on the material in between. We hope that the workshops organized in the final stage of the project are only a starting point of a longer programme of continuous professional development on the topic of IBMT. Our goal is to make basic concepts of IBMT precise, discuss drawbacks and benefits supported by instructive examples and evidence, and provide useful tips on classroom management to address teachers' concern about time.

Data gathered for RQ6 shows that teachers are very interested in the topic of assessment, so while preparing MERIA workshops this was taken into account along with the consideration how assessment might be complex and have various negative influence on the learning potential brought forward by the project.

Project impact on teachers in associated schools

Report on the second semi-structural interviews

Introduction

Interviews were held during July 2019. In total, there were 16 teachers interviewed (6 in Croatia, 2 in the Netherlands and 7 in Slovenia). Teachers were interviewed on the premises of their school or via WhatsApp. Teachers have been informed about the purpose of the interview and at the beginning of the interview gave their consent to being recorded.
As for the first round, all interviews were recorded and based on the data, interviewers wrote national reports. National reports were gathered and discussed in terms of conclusions for the project. We first present the national reports and then the conclusions on the level of the project.

Slovenia

Prepared by: Mateja Sirnik and Sonja Rajh, National Educational Institute, Slovenia

Teachers' background

Seven math teachers participated in the interview. All of them were involved in the project for two years and attended MERIA workshops and the MERIA Final Conference. Teachers were interviewed individually by Sonja Rajh, Mateja Sirnik and Mojca Suban. The interviewers marked the most important answers and summarized the findings.

Inspiration

Questions: *What is the most important thing you take on board from the MERIA project? What other ideas and activities do you take from the MERIA project? What activities from the MERIA project inspired you? How? Would you advise your colleagues to participate in the MERIA-project (or a similar one)? Why?*

The teachers were very pleased to participate in the international project MERIA, because they learned a lot about learning and teaching mathematics, and linked the theoretical knowledge of DTS and IBMT with practice. They were given the opportunity to try different ways of teaching, thus gaining a new perspective on mathematics teaching. Teachers gained greater insight into students' knowledge and abilities. During preparation of the implementation of scenarios, they worked on how to organize lessons for students to find a solution themselves and be active. The knowledge gained in this way is more permanent. Through scenarios, they learn the meaning and usefulness of mathematical content.

MERIA-activities

Questions: *What were your experiences at the MERIA-workshops? What were your experiences at the MERIA Final Conference? Through the MERIA-activities, did you find the sort of training/experience you were looking for? Did the MERIA-activities support you in bringing IBMT to practice?*

MERIA workshops highlighted a strong connection between theory and practice. Teachers were given specific instructions on what and how to change in teaching. They have shared experience in testing MERIA scenarios. This is very important to them as it contributes to the personal and professional development of the teacher and, consequently, to the improvement of the whole learning process.
Positive energy and work enthusiasm were noticed also at the Final conference in Zagreb. It was interesting to meet teachers from other European countries and talk to them about lessons, compare school practices, even break stereotypes. The teachers have gained insights into teaching mathematics in other countries. They were enthusiastic about the public session on the content and the implementation of the scenario, which was carried out in a class during the conference.

MERIA-outputs

Questions: *Did you read/use the Practical Guide or are you planning to? Why? Did you read/use the scenarios or are you planning to? Which ones? Why? Did the MERIA-outputs support you in bringing IBMT to practice?*

Teachers read all the translated scenarios, modules, and the Practical guide for inquiry-based mathematics teaching. They often reviewed individual sections of the practical guide. Each teacher carried out at least three scenarios. The materials were really helpful in introducing IBMT into their practice. They would like even more materials on the topic. The scenarios provide them a starting point for thinking about preparing their own scenarios for learning and teaching through inquiry.

Implementation

Questions: *What taken from the MERIA-project have you implemented in your lessons? Did that work? Did it bring what you hoped for? Did you experience any obstacles in the implementation? How did your students respond to your MERIA-related implementations?*

Teachers should take time to prepare themselves for the implementation of the scenarios properly. They have to follow the steps and allow students to work independently at their own pace and not interfere with their work or encourage them to divert their thinking according to teacher´s expectations. The teachers did not have any difficulties during the implementation. Some students find it difficult to develop higher levels of knowledge. They are afraid of researching because they lack experience.
The students were motivated to work and would like even more similar activities. The teachers can always find opportunities for such lessons, they just need to be flexible and open to new ideas. Later they need less time for practising mathematical content, because the scenarios enabled students to better understand and integrate mathematical knowledge.

Theoretical knowledge

Questions: *What do you know about IBMT (inquiry based mathematics teaching)? What have you learned about IBMT from participating in the MERIA project? What have you learned about TDS (theory of didactical situations)? What have you learned about RME (realistic mathematics education)? Has the MERIA-project increased your teaching repertoire/toolbox, in particular with respect to IBMT?*

The project allowed teachers to deepen their knowledge on IBMT. They have gained holistic insight into TDS, RME, and implementation of these approaches into the teaching practice. They report that they certainly know much more about these theories than they did before the MERIA project.
Due to the projects in which we participated before MERIA, some activities were performed unconsciously in accordance with TDS, IBMT or RME. The knowledge and experience gained from the MERIA project has given the confirmation that this type of teaching is effective and results in permanent knowledge. The acquired knowledge of TDS and RME allows teachers to not only limit the lessons on solving routine tasks, but also encourages them to choose learning situations in which students learn actively, integrate their knowledge with the new one and collaborate in more active forms of work.

Change

Question: *Has the extent to which IBMT plays a role in your teaching changed consistently? Do you think there should be more IBMT in the curriculum (national level), in your school, in your own classroom practices? Why? To facilitate more IBMT, do you think there is need for structural change on these levels? What changes?*

The percentage of lessons in which IBMT plays a key role has increased. Teachers believe that this type of work cannot be spread by imposition, but by sharing good practices, cooperation with educational institutions that educate future teachers, etc. However, some teachers think that a few lessons should be included into the curriculum due to the systematic introduction of IBMT, because many times such important things are left out due to time constraints.

Teacher´s views on IBMT:

• I think that only a few teachers know IBMT and that it is not emphasized enough in the curriculum. I and my colleagues at school should include more IBMT into out teaching practise.

• I would point out that in my previous teaching practice "the teacher explains how to solve the problem and the students try it out on at least one example" has changed.

• Permanent changes and systematic inquiry based teaching of mathematics cannot be achieved overnight. It is necessary to change the mindset of students, teachers and the way they teach.

• During the implementation of scenarios, all students participated actively. Although we thought that we are wasting our time during math lessons, it has shortly turned out that this seemingly lost time is three times recovered, as evidenced by good results when assessing knowledge and increased interest in the subject in question.

• I prefer problem-based approach in teaching to frontal lessons. I would like to implement more collaborative learning in small groups.

• Changes are necessary. We should educate children to perceive knowledge, skills and abilities as a value. Our teaching practise should include collaboration, reasoning, presenting findings, outcomes…

Croatia

Prepared by: Matija Bašić and Željka Milin Šipuš, Faculty of Science, University of Zagreb, Croatia

Teachers' background

In Croatia, around 12 teachers from associated schools have participated in the workshops on TDS and RME from September 2017 until November 2018 presented by project members at the Faculty of Science in Zagreb. They have all been interviewed at the beginning of the project. Among them, six were chosen for the interview at the end of the project because they were attending the majority of workshops and tested MERIA scenarios in their draft versions. This might rise scepticism that these six teachers were more interested in the project and hence biased, but we know that other teachers did not attend the whole program for personal reasons (they stopped teaching at school, reasons of personal character like illness of a close family member, involvement in other activities like design of the national matura exam etc.)
Final interviews were held on 27th June 2019 with three teachers from XII. Gymnasium in Zagreb, one teacher from Electro-engineering school in Varaždin, two teachers from Business school in Varaždin. The teachers were interviewed individually by Matija Bašić or Željka Milin Šipuš. Three teachers from Zagreb were interviewed at the Faculty of Science in Zagreb, while three teachers from Varaždin were interviewed at the Electro-engineering school in Varaždin. All interviews were held in a very pleasant atmosphere and lasted for approximately half an hour. The questions were posed one by one, sometimes with a clarification or a sub-question, but in this report the answers are presented jointly for a certain topic. The answers in this report present the clearest and most direct statements that the teachers gave.

Inspiration

Questions: *What is the most important thing you take on board from the MERIA project? What other ideas and activities do you take from the MERIA project? What activities from the MERIA project inspired you? How? Would you advise your colleagues to participate in the MERIA-project (or a similar one)? Why?*

Teacher A: She is more aware that mathematics education should include inquiry activities along with other teaching methods. The biggest impression on her was made by the implementation of the Area scenario. It was the first scenario that she has tried in her classroom and she found the situation very good, manageable and their students came to some conclusions on their own. She would recommend MERIA scenarios because she thinks discovery and investigation are useful/important.

Teacher B: Project MERIA was a positive new experience for her, it provided a new way of teaching, it was interesting to students and she appreciated discussing with other teachers. Scenarios bring a completely different way of teaching and she believes that it would be better if more people would get involved as that would lead to a more positive attitude towards mathematics.

Teacher C: As the most important thing she has learned how to introduce inquiry-based mathematics teaching (she calls it "problem teaching"). Before the project she did not implement such lessons. She would suggest such projects to other teachers because she notices that such teaching is encouraged by the new curricular reform in Croatia.

Teacher D: She is very satisfied with the project and she respects the project team ("I have expected a lot when I saw who is involved and I have received a lot"). She finds very useful to have colleagues present during the lesson and to receive feedback.
For her, the most important idea from the project is to engage students to work and discover on their own, as well as to show that it is acceptable to make mistakes. She needed encouragement from an authority to let students work autonomously and she claims that the project helped her how to achieve this. In particular, in a class where she has tried multiple scenarios, she noticed that with each new scenario she was more satisfied as students' reactions were more confident and had more "aha! moments". She would recommend the project to colleagues for these reasons.

Teacher E: She values new experience and seeing the experience of other teachers.

Teacher F: He finds IBMT ("such tasks") as something good, in particular as it encourages team work and makes teaching more dynamic. Later in the interview, he emphasizes the influence of IBMT on students' motivation. He claims he has improved his teaching skills and IBMT lessons helped him to connect better with the students. The biggest gain for him were MERIA scenarios and sharing the experience with other teachers.

Looking back at H5. Teachers feel that IBMT brings some additional positive outcome compared to the usual approaches and that it is worth the effort.
Still supported. Interviewed teachers show that they value inquiry activities and through implementation of the MERIA scenarios they have gained their own experience of IBMT. The implementation showed that their students can surprise them in a positive way and that the scenarios provide useable materials that support IBMT.

Evaluation of overall participation in project MERIA.
Teachers involved with the project MERIA express satisfaction with teaching materials (scenarios), professional development (in particular the aspect of sharing the experience with other colleagues and working with the project team) and the reactions' of their students to implemented lessons. Their involvement helped them to find a way to realize IBMT in their own classroom.

MERIA-activities

Questions: *What were your experiences at the MERIA-workshops? What were your experiences at the MERIA Final Conference? Through the MERIA-activities, did you find the sort of training/experience you were looking for? Did the MERIA-activities support you in bringing IBMT to practice?*

Teacher A: It was very useful that we were put in the position of the students and that during workshops we have seen how to work with our students. She found very useful to get feedback from other colleagues (teachers) and the involvement in the project motivated her team to discuss in the school about different issues during implementation of the scenarios. She is satisfied with this type of professional development, in particular because she has an authority (the project team) standing behind the material and providing arguments from research about the advantages of inquiry-based mathematics teaching.

Teacher B: It was positive that teachers from different schools were involved because they could compare similarities and differences. As a form of professional development, she finds MERIA useful and different from all other opportunities. Usually, teachers are passive listeners and it remains up to them will they try anything new and there is no feedback or discussion with the colleagues.

Teacher C: At the beginning of the project she doubted that the problems are too hard for her students, but she changed her mind after the implementation. She appreciated that one scenario (Braking distance) was worked out during the workshop and she says would not dare to implement it her classroom otherwise. She was satisfied with the workshops.

Teacher D: She finds great that the teachers have tried the scenarios and exchanged experiences as part of the workshops. She attended the Final Conference and finds it very nice, in particular the public lesson and the workshops in which teachers presented their own ideas. She liked the organization of the presentations so that teachers had to visit presenters at different stations, and she intends to use that idea for future workshops. She finds the activities of project MERIA much better than the opportunities she has at the national and county level because there she only had theoretical lectures, while in MERIA teachers were active participants.

Teacher E: She liked that the teachers went through the scenario as students.

Teacher F: In the beginning of the project he did not know what to expect and he had a fear that his school will not fit in the project. The workshops and his implementations of the scenarios showed that it was good to join the project. He finds that is easier to improve teaching skills in direct contact (live workshops)

than by reading or following on-line lectures. He participated in the Final Conference and that was his first international professional experience. He appreciated the most exchanging experiences with other teachers (seeing they share the same problems) and seeing others' ideas. He grades the professional development he has gained in MERIA project above expectations. He had to prepare in detail for the implementation of the scenarios and that required extra time, but both him and his students are quite satisfied, so he would gladly join in similar initiatives again.

Looking back at RQ4: What kind of training and material teachers are interested in?

Teachers appreciated live workshops in which they have an active part. They emphasize the following situations as beneficial for their professional development: playing out a scenario in the role of a student, implementing a scenario and discussing the implementation with other colleagues, receiving detailed materials and ideas from others. The impression about the materials changes from scepticism to appreciation through trying out scenarios as students (in the workshops) or with their own students. It appears that the main criteria for the materials are manageability (understanding of the rationale behind the scenario and the confidence to implement it as it is intended) and students' reactions.

Evaluation of MERIA workshops and MERIA final conference.

Teacher F and Teacher D are the only two teachers that participated in the MERIA Final Conference. They had a very positive experience of sharing ideas. The interviewed teachers did not participate in the official MERIA workshops (January-April 2019), but they had similar activities and attended more meetings (September 2017 – November 2018) where teachers discussed among themselves and with project partners about the implementation of the scenarios. The professional development provided in MERIA project is graded above expectations due to many opportunities for active participation.

MERIA-outputs

Questions: *Did you read/use the Practical Guide or are you planning to? Why? Did you read/use the scenarios or are you planning to? Which ones? Why? Did the MERIA-outputs support you in bringing IBMT to practice?*

Teacher A: It is nice to have a paper version of the Practical Guide. She thinks it has a lot of information which is systematically presented. Concerning scenarios, she feels that it is not easy to work on someone else's materials, but that the scenarios are well designed. She also trusts the materials because of the authority of the authors, but also claims that the scenarios cover her interests and difficult topics. She feels supported in implementing IBMT both by scenarios and workshops. She tried out the Area enlargement and Bicycle factory.

Teacher B: She does not remember the details from the Practical Guide, but she recalls it was a starting point for the work on the project. She has tried out the Abba scenario and Spreading the rumour (a scenario that was not published in the end but was tried by teachers in the testing phase). She mentions certain

ambiguities in the draft version of the scenarios, but for the final versions she thinks they were based on good ideas and very detailed. She would try out more scenarios if she managed to prepare materials. She is aware that it takes a lot of time to design materials properly.

Teacher C: She has read the Practical Guide and finds that it is useful to have many examples that she can use as a teacher either as a ready material or a source of inspiration. She has implemented the Braking distance scenario.

Teacher D: She finds the Practical Guide useful to transfer the ideas to other colleagues. She has tried out five scenarios: Area Enlargement, Bicycle factory, Braking distance, Spreading the rumour and Abba. She finds that all the scenarios are detailed, useable and more than she expected. Although the scenarios are easy to use, one needs to prepare well for the lesson and workshops have also helped her a lot in preparation.

Teacher E: She did not read the Practical Guide. She liked that she received concrete materials. She has tried one scenario with her class, the Salary. For her it is important to have concrete guidelines.

Teacher F: He considers that the Practical Guide is of help in preparing his lessons and returns to examples in it regularly. He has implemented three MERIA scenario: Area enlargement, Bicycle factory, Achilles and the turtle (the last one is not one of the five published scenarios, but a draft was prepared by the project team and tested in the associated school). He works in a business school and he thinks his students need easy problems or additional hints. He decided to modify certain scenarios in terms of presentation of the problem and working materials. He limited the materials not to confuse the students with too much choice or developed Geogebra applets with additional hints/tools that can guide weaker students.

Looking back at H2. Lack of adequate system support, teaching and learning material, teacher training and overloaded prescribed program are main reasons for not applying IBMT approach in teaching on regular bases.
Still partly supported. At the beginning of the project teachers were not so familiar with IBMT and therefore did not miss having support or materials for IBMT. Teachers did mention the overloaded program in this context.
Now, after MERIA, teachers mention that the problem of not having enough time to prepare for IBMT lessons still exists, but they have the time and the will to include a few of such lessons during the year. MERIA scenarios and the theoretical ideas underlying them support teachers in preparing and carrying out IBMT lessons. In particular, MERIA scenarios fit in with the curriculum, so the scenarios can be part of the regular lessons.

Evaluation of MERIA scenarios and MERIA Practical Guide.
Teachers have read parts of the MERIA Practical Guide and find that it is useful to have an explanation of the theoretical ideas and many examples that they can return to in the future. Scenarios are accepted with positive reactions because

they provide situations that can be dealt with on different levels, so they are useable for working with students of various abilities.

Implementation

Questions: *What taken from the MERIA-project have you implemented in your lessons? Did that work? Did it bring what you hoped for? Did you experience any obstacles in the implementation? How did your students respond to your MERIA-related implementations?*

Teacher A: She has picked up many ideas and concrete examples. Implementation of the scenarios did not go completely along with her expectations as all the students have not reached the target knowledge, but for her it was important that the students worked on their own. It takes time for the students to adjust as they did not know what is expected of them and how to phrase their answers (e.g. they would just say that the area increases and not specify how), so she felt that she needed to help. Students overall reacted positively, and she thinks that implementing such activities in continuity would make clear that it is not all about the result, but about the process of exploration, too.

Teacher B: At first, her students were completely lost. Better students did not know what was expected and they were afraid of making a mistake. Students who are usually less active, but not shy, showed higher engagement than usual. With time students accepted that they can also learn from mistakes. Some students only waited, but after some time also started working. She does not expect that all students reach the desired result, but she is satisfied that the great majority of students was "on the right track". She elaborates on the dilemma which approach brings long-term benefits and concludes that inquiry based approach is more useful. She is constrained by the yearly plan and an obligation to give many grades to all students. Also, she emphasizes the attitudes of students and parents as an obstacle to introduce more changes. She guesses that parents would think that students were only playing and had not learned anything, while she thinks students would work more during inquiry based lessons than while passively copying solutions of tasks. She saw that during the implementation of MERIA scenarios, students participated actively, thought about mathematics and reasoned. It was certainly fun for the students as they talked about it at home and to friends in other classes. Students were engaged in particular in the Abba scenario because of an element of mystery and it was very positive for the teacher that the students realized that behind a guessing game there is no magic, but a mathematical (scientific, logical) explanation.

Teacher C: As mentioned already, she was skeptical that her students will not understand the problems and will not be able to work on them autonomously. Her students were very active and engaged and reported that they would like to experience more similar lessons. She was satisfied. As an obstacle she mentions that she needed to adapt to a new style of teaching, in particular it was hard for her to refrain from guiding the students all the way as she feared that they will not work in the "right direction". She notices that in the end the students accepted that they should work on their own. The scenario fitted well with the program she teaches (she used it as an introduction to the quadratic function), although she usually uses only 15 minutes instead of 90 minutes to introduce a topic.

Teacher D: As mentioned earlier, she tried out multiple scenarios with the same class and she noticed that students' expectations adapted with time (when asked to gather their desks as in the first activity, students asked "Shall we write a letter to the mayor again?"). She finds that MERIA scenarios brought new ways of presentations that are interesting to students: writing a letter about the obtained results or multiple students/groups writing their conclusions on the board at the same time. When she was carrying out one scenario, she formed a group of the "best" students in the class and noticed that they have done "what they were expected". Also, she formed a group of the "worst" students and was positively surprised by their work. As an obstacle she first mentions lack of self-confidence and the fear of making a mistake. She made a conscious decision not to use ICT, but she will also try the same scenarios with ICT in the future. Now that she has the experience of the scenarios, she sees that she can incorporate such lessons at least once in four months ("a semester"). She also adds that these lessons would lose their point if done too often.

Teacher E: Her students are used to get direct instructions what to do, but in the end they were happy that they came to some conclusions on their own. As main obstacles she sees students' lack of prerequisite knowledge and negative attitude towards mathematics in her school. In general, she feels limited by the external evaluation ("matura"), but she explains that logical thinking is required at that exam and that its development is supported by MERIA scenarios. As an example, she mentions that graph of a quadratic function is important piece of knowledge and that the scenario Braking distance enables students to understand it better.

Teacher F: His students are not motivated to study mathematics, they struggle with self-confidence and prerequisite elementary mathematics (e.g. fractions, elementary algebra). He has implemented three different scenarios and noticed that students accept working on their own after multiple IBMT lessons based on MERIA scenarios. For the Achilles and the turtle scenario he designed an animation to present the problem more clearly and that helped to get higher engagement of students. His students struggled to formulate general conclusions, but he did not expect them to do that as they were not given such tasks before. He did not consider that as a failure because he could evaluate from further questions that the students understood the principle that was aimed (e.g. he asked what happens to the area when a side is 4 times bigger and the students reply that the area gets 16 times bigger).

Looking back at RQ2: What other obstacles for not innovating teaching and learning process can be recognized?

Teachers often base themselves on the textbooks. Now, they indicate how MERIA is offering opportunities and inspiration to have lessons not based on the book. Still, time to prepare for those lessons is difficult to find. Also, using computers in classroom can be very difficult, but with parts of the MERIA scenarios teachers had a good experience using computers in classroom. Some teachers report their students' lack of prerequisite knowledge (e.g. working with fractions or elementary algebra) as the main obstacle for students' autonomous problem solving.

All teachers talk about the need for them and the students to change their behaviour and expectations ("the didactical contract"). Teachers find it hard to refrain themselves from helping the students, but notice that they get positively surprised when they do. Students at first struggle without guidance ("what am I supposed to do?"), but quickly adapt. Students' successful adaptation is clearly evident in classrooms where multiple MERIA scenarios were implemented. Students' reaction on MERIA scenarios were mainly positive and the teachers find the materials useful.

Theoretical knowledge

Questions: *What do you know about IBMT (inquiry based mathematics teaching)? What have you learned about IBMT from participating in the MERIA project? What have you learned about TDS (theory of didactical situations)? What have you learned about RME (realistic mathematics education)? Has the MERIA-project increased your teaching repertoire/toolbox, in particular with respect to IBMT?*

Teacher A: She finds that she likes the concept of "discovery" and thinks it is important. She thinks that the attitudes of students and parents should change. (Further elaborated in the next topic).
She has learned how to organize a lesson and institutionalize what was achieved during the lesson. She does not use the vocabulary of TDS or RME and could not formulate what she has learned about or from these theories. Nonetheless, she appreciates situations that put students in the active role (TDS) and various types of problems (RME). She adds that the problems do not have to come from the real world.

Teacher B: She characterizes inquiry based teaching by students' active participation of and discovery, while the teacher leads the process. She believes it is a more useful approach than traditional lecturing. She Also, she likes changes because novelties engage students. During the project, she has made conscious how much she leads the students and that students are capable to make conclusions by themselves to greater extent than she thought before. She explicitly says that she does not use terminology from the theoretical frameworks with her colleagues, but describes phases of TDS in great detail. While describing formulation and validation, she emphasizes the benefits of feedback coming from peers instead of the teacher. In her class, students' presentations developed in a positive atmosphere and without students making fun of each other.

Teacher C: She learned how to implement inquiry-based teaching, in particular the different phases of a lesson. She will implement more scenarios.

Teacher D: Before engagement in the project, she thought that inquiry based teaching should involve a larger student's project that is done at home during a longer period. She points out the problem from the public lesson at the MERIA final conference as an example of a problem where she now recognizes a potential for students' inquiry. She defines inquiry based teaching as a situation in which students autonomously investigate and discover pieces of knowledge. She does not use the vocabulary of TDS or RME and could not formulate what she has learned about these or from these theories. She used to be sceptical to group work

as she feared that only one student in the group would do all the work, but after trying it as a part of MERIA scenario she changed her mind.

Teacher E: She defines IBMT as situations where students come to conclusions on their own. She sees the role of a teacher in supporting the students and intervening when they struggle ("not to tell them the solution but direct them if they are on a completely wrong path"). She did not learn about inquiry based approach in her studies, so she has learned a lot. She finds workshops activities in which teachers are put in the role of a student very useful as a preparation for the lesson. As a main tool she is taking from the project is the idea of incorporating small inquiries as parts of a lesson.

Teacher F: For him, IBMT is every form of a lesson in which teacher does not lecture all the time and students are more engaged than just answering to simple questions. He appreciates that style of teaching and notices it is a novelty in schools. He easily includes own examples in the discussion, for example when teaching analytical geometry, he poses a problem to determine the equation of a circle passing to three fixed points which he poses in the context of a map with treasure buried equally distant from three points. He elaborates on the potential to use this problem to link the situation with elementary geometry of perpendicular bisectors. He feels that he improved in implementation of "non-traditional activities" and he hopes he can use the structure of MERIA scenarios to design his own scenarios. From RME, he recalls the lectures from the Final Conference about the context in mathematical problems.

Looking back at H1. Teachers are aware of IBME approach and appreciate it but they do not implement it in classroom practice of mathematics teaching.
Not supported anymore. Teachers have a much more precise idea of what IBMT is and of possible ways how to implement it. In the beginning of the project they did not consciously implement IBMT in a structural way or with clear understanding. Now they have built experience trying out MERIA scenarios.

Change

Question: *Has the extent to which IBMT plays a role in your teaching changed consistently? Do you think there should be more IBMT in the curriculum (national level), in your school, in your own classroom practices? Why? To facilitate more IBMT, do you think there is need for structural change on these levels? What changes?*

Teacher A: She says that it was the first time that she has completely let students work autonomously. It is a change for her to have the whole lesson planned (with all possible outcomes in each part), as she did detailed plans only for smaller parts of the lesson. She thinks IBMT should be more incorporate in the curriculum and on the level of her school and classroom. She did not experience such activities as a student. She feels that textbooks dictate what type of problems she should do (long and boring tasks) and that she needs more courage for a change. She thinks that for the change to happen the expectations of students and parents need to be

negotiated. At the moment the expectation of parents is that she should be teaching many repetitive tasks from the textbook, while her conviction is that it is more useful to insist on understanding. As an example, she says that students know the procedure for solving linear equations but cannot explain why they "move" a term from one side of the equation of the other. "They know how to solve the equation, but they have no clue what they are actually doing." She thinks that external evaluation (final national exam – matura) is aligned with inquiry based mathematics teaching because the most difficult problems are not as textbook tasks and they are similar to the problems in MERIA scenarios. She thinks that time will not be a constraint because with experience on can plan IBMT lessons. Culturally, she likes that teachers observe work of their colleagues and that this should be promoted.

Teacher B: She sees the difference now as opposed to few years ago, at the beginning of the project. She is conscious of the structure of the scenario (different phases) and she is trying to incorporate more of her own inquiry based lessons although with more guidance than in the MERIA scenario. The main difference is that she lets students to think on their own for a longer time and does not see that as a waste of time. She thinks that one should not do inquiry all the time, but she needs more materials and scenarios. She also thinks that the yearly plan is too packed with content and that the students would benefit by a reduction of content. Furthermore, her teaching is dictated by the final national exam as students keep on asking "is this a part of the final exam?".

Teacher C: She is in favour of changes, but in small steps. One should not exaggerate with new methods and the automatization should be implemented in a different way. She considers IBMT as an enrichment and she was aware before the project that as a teacher she can use various materials (not only textbooks). She thinks that external evaluation ("matura") and inquiry-based teaching are not in collision, and that students learn with every teaching approach. She would support more changes, but she would also like to see the results of the experiments before big changes that would affect whole generations.

Teacher D: For the new school year she already plans to incorporate inquiry based lessons in each class. She supports changes and claims that teachers in her school that were involved in the project will improve their teaching methods. She is sceptical towards changes on the national level but is oriented to personal improvements and would like to be a part of innovative projects in the future, too.

Teacher E: She says that she has always tried to incorporate inquiry elements in her classes and she does not feel a big change. She claims that her students do not have a positive attitude towards mathematics and describes their expectations as "show me the procedure, give me another similar task and grade me". She struggles with engaging students in critical thinking and as a main obstacle for a change, she points to students' attitudes and the general culture in the society.

Teacher F: He wants to include IBMT lessons at least once or twice in a semester. He plans to use Bicycle factory scenario with the first grade. He thinks that the curriculum (the yearly program) is overloaded with content and that the yearly

plan should include project-based lessons. He mentions the problem of not having enough ICT support, but also that this is now changing. Finally, he notices that textbooks are not aligned with IBMT and he misses didactical materials with examples for programs in vocational and business schools (where the number of mathematics lessons is smaller compared to gymnasiums).

Looking back at H4. Teachers have a positive attitude towards changes in teaching and learning and have concrete ideas what and how to change on the macro (system) and micro (classroom) level.

Still supported. Interviewed teachers reflect on various levels of the educational system and how they affect their teaching. All of them share a positive attitude towards changes, although they have different viewpoints about the pace, extent and the form of changes. They report that they are already planning to use MERIA scenarios in the future, but they would like to have more scenarios or developed ideas. They do not consider external evaluation to be in collision with IBMT approach. They would reduce the amount of content in the curriculum and include project-based (inquiry) lessons in the yearly plans. On the macro scale, some teachers express the need for cultural change (students' negative attitude towards mathematics), which is beyond their reach.

Looking back at RQ7. What ideas teachers have for a change on a classroom level?

Teachers would dedicate more time to activities in which students take a more active part, but also limit the amount of such activities to a few times in a year. They all put forward their attitude that mathematics lessons should encourage critical thinking and students' autonomous learning to a greater extent that it is now. Also, they see benefits for improvement of their teaching skills from including colleagues to their classes or discussions about the implementation of lessons.

The Netherlands

Prepared by: Rogier Bos, Utrecht University, The Netherlands

Teachers' background

Final interviews were held with two teachers. The other two teachers that were interviewed at the beginning of the project, stopped their involvement with MERIA. One stopped as a teacher at secondary school level, while the other left the project for unknown reasons. The teachers were interviewed by Rogier Bos. During individual interviews only the interviewed teacher and the researcher were present. One teacher was interviewed over WhatsApp and the other at her work in a quiet room. All individual interviews were held in a very pleasant atmosphere and lasted more or less half an hour.

Inspiration

Question: *What is the most important thing you take on board from the MERIA project?*

Teacher A: The new ideas for lessons. MERIA inspired to structure my inquiry lessons in phases, some of them a-didactical. I took the idea of building on students work to introduce a concept. I like the ABBA-scenario and would like to try the Slide scenario, maybe also to introduce the subject of continuity. The Conflict lines-scenario I'm going to repeat this year, after trying it out last year. Generally, I picked up the idea of students discovering a concept themselves. I used this idea and let students discover the notion of concavity and convexity from examples and non-examples. I let students use pictures to explain their ideas. On the whole, MERIA inspired me to do something else with my lessons occasionally.

Teacher B: If I do an inquiry lesson, I take a moment after five minutes of the action phase to discuss the progress with the students, allow them to share their strategies; make sure everyone knows what to do. I use this little intervention much more often than just in MERIA activities. When I did lessons like this before, I would give hints to the weaker groups after a while, helping them solve the problem. Doing this for every group separately takes too much time. A shared moment after five minutes is more practical. It is not important that one group could come up as the cleverest one, I want all groups to make progress and work nicely. It is also useful that the strategies that do not work are shared. This might seem a small thing, but it helped me approach my inquiry lessons differently. I tried this structure in other activities. My colleague suggested that students like to discover that there is a mathematical short-cut solution to a problem. I tried this in the context of the trajectory of a ball. Students had to figure out when it would hit the ground again if the trajectory was a parabola described by a given equation. Students used all sorts of methods, e.g. with a table, but not decomposition of the equation, which they had just learned. When they later learned about this approach, they were very excited. In this group this subject goes very well now.

Looking back at H1. Teachers are aware of IBME approach and appreciate it but they do not implement it in classroom practice of mathematics teaching.
Not supported anymore. Teachers have a much more precise idea of what IBMT is. In the beginning of the project they did not consciously implement IBMT in a structural way or with clear understanding. Now they have built experience trying out MERIA scenarios. They apply the underlying ideas with their own scenarios.

Question: *What other ideas and activities do you take from the MERIA project?*

Teacher B: I really liked one of the workshops in the final conference, on statistics

and how you could use statistical data to trick people. The workshop leader had very nice examples.
The slide scenario is now a regular part of my lesson plan. I am also planning to do the Linear scenario. Generally, I want to download the scenarios, look at them and see how to fit them in my lesson plans. I don't want to forget about the scenarios, because I like a lot of them. I also really liked the problem in the Lesson Study during the final conference. It would be a nice problem for my more talented students.

Question: *What activities from the MERIA project inspired you? How?*

Teacher A: The Roping cutting activity from Britta. I already tried it out, with some adaptations.
The lesson study with 100 teachers was brilliant, both the setting and the problem students worked on.

Question: *Would you advise your colleagues to participate in the MERIA-project (or a similar one)? Why?*

Teacher A: Yes! I am trying to include my colleagues.

MERIA-activities

Question: *What were your experiences at the MERIA-workshops?*

Teacher A: It was useful to learn about MERIA and underlying theories. But the tempo was a bit low, and sometimes abbreviations were not explained. It was a bit disappointing that not as many of the participants actually tried out the scenarios. Most inspiring was being together with colleagues who are also enthusiastic to try new things.

Teacher B: What stuck most with me from the workshops is what I just mentioned before about not intervening and having a moment after five minutes to discuss work so far. I went home and tried the slide scenario the next day. It happened to be the day before we started with the derivative. I struggled a bit with collecting data from that lesson.

Question: *What were your experiences at the MERIA Final Conference?*

Teacher A: I had a wonderful time. It was interesting to learn about mathematics education in different countries. I was impressed by the developments in Croatia. The 15-minute workshops were great; I would have liked more of them (in particular more from Slovenia and Croatia). Talking about the scenario was great too, but it would have been nice to talk about all scenarios, since I tried also some of the scenarios that were not part of the "last five". Introduction speeches were a bit boring, but probably inevitable. I missed a little reminder on the goals and ideas of the MERIA-project.

Question: *Through the MERIA-activities, did you find the sort of training/experience you were looking for?*

Teacher A: I expected results that were easier to use, and more easily accessible. Not this long table, but something shorter.

Question: Did the MERIA-activities support you in bringing IBMT to practice?

Teacher A: Yes. I realized (1) I want to do this more often, (2) that I need to give students opportunity to work for themselves (keep my mouth shut), (3) TDS brought me lots.
I got a more nuanced view of RME. That in RME 'Realistic' does not mean 'taken from reality', but 'meaningful' possibly in a mathematical context. I realized that RME can be part of my toolbox. You don't have to be dogmatic about one theory or the other. My impression of mathematics education research has improved.

MERIA-outputs

Question: Did you read/use the Practical Guide or are you planning to? Why?

Teacher A: I think I should have read it and it would have brought me lots. It must be somewhere on my shelves.

Teacher B: I have read it, although I didn't study it thoroughly! I liked to see how the scenarios were incorporating these ideas.

Question: *Did you read/use the scenarios or are you planning to? Which ones? Why?*

Teacher A: Yes. Diagonally, to choose which ones to try

Question: *Did the MERIA-outputs support you in bringing IBMT to practice?*

Yes. By trying out.

Looking back at H2. Lack of adequate system support, teaching and learning material, teacher training and overloaded prescribed program are main reasons for not applying IBMT approach in teaching on regular bases.
Still partly supported. At the beginning of the project teachers were not so familiar with IBMT and therefore didn't miss having support or materials for IBMT. Teachers did mention the overloaded program in this context.
Now, after MERIA, Teacher A mentions the problem of having enough time to prepare for IBMT lessons still exists. Teacher A and Teacher B both mention how the MERIA-scenarios and the theoretical ideas underlying them support them in preparing and executing IBMT-lessons. In particular, Teacher B mentions how the MERIA-scenarios nicely fit in with the curriculum, so the scenarios can be part of the regular lessons.

Implementation

Question: *What taken from the MERIA-project have you implemented in your lessons?*

Teacher A: ABBA, Doubling-formulas, Conflict lines

Question: *Did that work? Did it bring what you hoped for?*

Teacher A: ABBA did. Conflict line too, although it was a bit too easy for my group. Doubling formulas was more difficult, because the context did not give support in approaching the problem. Still students worked well.

Question: *Did you experience any obstacles in the implementation?*

Teacher A: Getting the time scheduling sorted. The scenarios were such a long read.

Teacher B: I experienced how the groups size can really matter. Groups of two worked better than groups of four. I tried the Gossip spreading scenario, but I hadn't formulated the problem precisely enough. I hadn't thought enough about what the formulation should be after five minutes. So I realized I do not need to prepare properly for an inquiry lesson. The second time I did the lesson, it went a lot better.

Looking back at RQ2: What other obstacles for not innovating teaching and learning process can be recognized?
In the beginning of the project Teacher A mentioned that she bases herself too much on the book.
Now, she indicates how MERIA is offering her opportunities and inspiration to have lessons not based on the book. Still, time to prepare for those lessons is difficult to find.
Teacher A also mentioned that using computers in classroom can be very difficult, but as part of the MERIA-scenario Conflict line she had a good experience using computers in classroom. Teacher A also mentioned that her school joined in the Primas Project. She did not find it attractive to go to the website and search for materials. This should be a concern for MERIA.

Question: *How did your students respond to your MERIA-related implementations?*

Teacher A: Fine. They are used to doing something else, like mathematical thinking activities and puzzles. They enjoyed it. They appreciate it when there are lessons/activities not based on the book. Occasionally a students asks why we have to do this or if it is going to be assessed.

Theoretical knowledge

Question: *What do you know now about IBMT (inquiry based mathematics teaching)?*

Teacher A: "It is important to have a good problem or question, to give direction. That students have the opportunity to discover (no secret hints). Sometimes they find what you want, sometimes not. As a teacher you need to value students' findings and use them to give instruction on the learning goals. IBMT begins with a good context, a good problem to study a concept. I think students then more easily remember the concept, since they feel ownership. It does costs a lot of time. I see IBMT as something you do sometimes, not all of the time. But it would be great if there are ideas for IBMT across the whole curriculum; just to have the opportunity.
Teacher B: I'm a bit slow with these things. I think some of the exercises in this book are based on RME ideas. But to make it work I have to make sure I make something out of it.

Question: Has the MERIA-project increased your teaching repertoire/toolbox, in particular w.r.t. IBMT?

Teacher A: Yes, (1) it offered concrete ideas, (2) it gave inspiration to develop new ideas (3) it offered better theoretical support, in particular through the phased structure of TDS.

Teacher B: What I liked about MERIA (better than other courses) is that I have many concrete tools to works with now. The materials fit in with the curriculum. This makes them very applicable. You don't have to go and find extra time to do it; you can really fit it in with your regular lessons. And it is clear where you have to fit them in the curriculum.

Looking back at RQ4: What kind of training and material teachers are interested in?
Before the MERIA-project Teacher A said that she would like to have help implementing IBMT in classroom. She said that teacher trainings are often either to low level or they are unrealistic in what is achievable in classroom. Teacher A has clearly picked up a lot from being involved in the MERIA-project. She indicates she did not use the practical guide, so most of it must come from workshops and lectures. These workshops, including the homework (trying out scenarios), seem to have achieved the goal of supporting Teacher A achieving a sustainable implementation of IBMT in her lessons, through providing concrete materials, experience and a theoretical framework.

Change

Question: *Has the extent to which IBMT plays a role in your teaching changed consistently?*

Teacher A: We were already in a change process, but MERIA has stimulated me to continue.

Question: *Do you think there should be more IBMT*
- *the curriculum (national level),*
- *in your school*
- *your own classroom practices*

Why?

Teacher A: Yes, yes, yes. Maybe it is difficult to assess on a national level. But perhaps that should not be the goal. I think IBMT stimulates students' learning to think very well. That's what I want to work on with my students, and what I wish for other students as well…as an addition to traditional education. I think IBMT fosters students' independence. Makes them better problem solvers, which is important to function in the 'real world'; outside the classroom problems are not pre-structured, so it is not clear what mathematics you need to apply. Maybe not all students realize this themselves yet, because they haven't been in this sort of situation outside school too often. But that need not matter; they can learn inquiry trusting they will need it later. I learned a lot of things when I was at school, merely trusting I would use it later. It would be great if students see the benefits of inquiry learning, but otherwise they can consider it an extra in the curriculum.
Teacher B: I already did a lot in my classroom, but now I now I know how it fits in with theories. I have more structured ideas about it and more examples available.

Question: *To facilitate more IBMT, do you think there is need for structural change in*
- *the curriculum (national level),*
- *in your school's organization or*
- *your own classroom practices*

What changes?

Teacher A: In school we need time and money. Workload is high, and IBMT costs time. I find IBMT important, but I am not sure about priorities. If we have time and money, there are many things we could do. We did get some time this year and we used some of it on developing inquiry-like tasks.
At my school the climate for trying out new things, like IBMT, is good.
I was intrigued by Britta talk about the future of the talk and the possibilities of Lesson Study to implement IBMT. I was surprised about how many different observations came from observing a lesson with so many teachers. After three minutes I thought all had been said, and then that turned out to only be the beginning of it.

Teacher B: My mathematics colleague did not like IBMT. But I did one inquiry lesson with a physics teacher and she witnessed it and saw that there are some yields to it; that it brings a different flow to the classroom. In our school we work with domains and teachers share classrooms, so sharing experiences is default.
I think the Netherlands is ahead of other countries with this. Our central exams have Mathematical Thinking Activities, but also tasks where a lot of data needs to organize, before you can apply mathematical procedures. I don't think the exam should have more non-routine tasks than it now has. Inquiry is more suitable for classroom activities.

Looking back at H4. Teachers have a positive attitude towards changes in teaching and learning and have concrete ideas what and how to change on the macro (system) and micro (classroom) level.
Still supported. Teacher A wanted a fuller curriculum (with more time) and more time to experiment, in particular with Mathematical Thinking. MERIA gave her the opportunity to experiment with inquiry, which is closely related to Mathematical Thinking. She expresses that she finds IBMT important, but among other things. She sees a great future for implementing Lesson Study at Dutch schools.
Teacher B experienced how a colleague was more positive about IBMT after experiencing it. Obviously, Lesson Study would provide a great way for a group of teachers to acquaint themselves with IBMT. On a national level, Teacher B thinks, there is enough attention for IBMT, through the implementation of Mathematical Thinking in the national exams and all the attention that got. Teacher A thinks this assessment of Mathematical Thinking is a bit problematic and most change should happen *in the classrooms*; she wishes all students to learn to think in their mathematics lessons.

Conclusions

Prepared by: Matija Bašić and Željka Milin Šipuš, Faculty of Science, University of Zagreb, Croatia

We have interviewed six teachers from three associated schools in Croatia. All of them participated in a cycle of professional development from September 2017 until November 2018 organized by the Croatian project team members. These workshops were different from the MERIA workshops organized in the beginning of 2019 as project's methodology and results were under development. Nonetheless, all of them first participated in lectures on basic tenets of IBMT, TDS and RME, then experienced at least one MERIA scenario and discussed the implementation with others at the final meeting of the cycle. Two teachers (Tatjana and Ivan) participated in the Final Conference. All six teachers are familiar with the Practical Guide, but have not seen MERIA modules as these were in part written based on their experience. It is clear from the interviews that teachers evaluate MERIA scenarios as detailed and useable and are satisfied with students' reactions.

Teachers appreciate that they have received detailed materials and an opportunity to discuss the implementation with other colleagues. From the materials they value the structure as it provides them the way how to implement inquiry based teaching, while feedback from other colleagues helps them to gauge their expectations and improve their actions. Teachers' understanding of inquiry based teaching is mostly experiential and they define it as "students working on their own". This understanding has certainly evolved through workshops as they report that their perspective has changed, and they provide examples on it. Concerning their own skills, teachers have improved in their role to "withdraw in a meaningful way" and value the time given to students to work without their interference.

Teachers claim that they would like to share their knowledge with other colleagues and introduce them to inquiry based teaching, but it is our impression that it might be still very difficult for teachers at this stage of the project to move on from their role as teachers to teacher trainers. On the other hand, teachers clearly illustrate their values and support their reasoning with positive examples. Teachers have not adopted the vocabulary of the presented frameworks, but it is clear from the interviews that they use the concepts that are central to these frameworks. For example, teachers emphasize students' autonomous actions ("adidactical situations"), they often mention students' expectations and time needed to adapt to a novel way of learning ("didactical contract"), they are aware of the structure of a lesson ("phases in TDS") and of their role to support students ("scaffolding") and to connect students work to the topic of the lesson ("target knowledge" and "institutionalization"), also they value realistic and open problems ("rich contexts in RME" and "didactical potential"). As a form of professional development, they rate MERIA workshops (and the project in general) very positively and emphasize that they highly appreciate being active participants and discussions about their experiences. They point out that there are a few such opportunities as mostly professional development is provided in a form of lecture with no materials.

We have teachers from different types of schools. Some teachers identify their students as "not so good as in other schools" and they find students' lack of prerequisite knowledge as an obstacle to advance to further topics and experimenting with different types of teaching. Some students show negative attitudes towards mathematics and persisting resistance to engage in a discussion or to think about the problems autonomously. Teachers report that students "learn for grade" and describe students' expectations as "show me the procedure, give me another similar task and grade me". Also, all teachers reported that during the first implementation of a MERIA scenario students were confused by the change of the teaching method and asked for help what to do. Those teachers that tried more scenarios in the same class report that students adapt successively with time and that there was a high level of satisfaction of both students and the teacher. Teachers claim that their teaching is heavily constrained by the final national external assessment ("matura") and the amount of the mathematical curriculum it covers, but they clearly point out that logical thinking is also

required at these exams and that MERIA scenarios support its development. So, it is useful to incorporate inquiry based teaching since it supports mathematical reasoning required in the final assessment, but this argument is not always clear to students and their parents. This all points that the negotiation of the didactical contract is crucial for successful implementation of inquiry based teaching.

Almost all teachers discuss students' work in terms of successful solving of tasks ("on the good path", "lost", "did what I expected" etc.) as though there is only one way of solving the problem. Teachers express that they value different approaches and that they are encouraging students to make mistakes and learn from them, but they know with which achievements they are satisfied (which is not necessarily the ultimate target knowledge as written in the scenario) or when they are positively surprised. It seems that teachers have an understanding of benefits of inquiry based approach and a clear set of personal values concerning education.

Teachers feel that inquiry situations bring more dynamics to their classrooms and that MERIA scenarios are interesting to their students. Teachers emphasize the attitudes of students and parents as an obstacle to introduce more changes. They suggest that parents would think that students were only playing and had not learned anything, while they think students work more during inquiry based lessons than while passively copying solutions of tasks. Students get more engaged because of an element of mystery and it was very positive for the teacher that the students realized that behind a guessing game there is no magic, but a mathematical (scientific, logical) explanation. In particular, students like new ways to present their results (e.g. in the form of a letter to a mayor, or when multiple students write on the board at the same time) and working in a relevant context (discuss payrolls, provide advice, construct a slide). Overall, all teachers report that students were engaged and that they were positively surprised by the students with more modest achievements. Some teachers are planning to incorporate MERIA scenarios and their own similar inquiry lesson in their yearly plan. In general, teachers feel that they have enough time and freedom to incorporate inquiry based lessons during the year and add that this statement is related to the fact they have tried the scenarios ("now that I know what it is about") and received positive feedback ("my students managed and were satisfied"). So, we conclude that it is personal experience that leads teachers' decisions. It is also important to note that teachers express positive attitudes towards the materials because they trust the authors (the authority of the project team). Hence, we further hypothesize that the use of materials and the success of the professional development also depends highly on the persons providing it.

Evaluation of MERIA scenarios – teachers' perspective

Prepared by:
Kristijan Cafuta and Selena Praprotnik, University of Ljubljana, Slovenia
Analysed by: Matija Bašić, Faculty of Science, University of Zagreb, Croatia

MERIA scenarios were tested in class. In the first round, answers were gathered after the implementation of the pilot scenario Area enlargement in all four countries. This facilitated the project team to develop guidelines for development of other scenarios. In the second round, there were 11 different scenarios in total and we used the results together with feedback from the teachers during workshops to decide which five scenarios to finalize. In both rounds, students completed questionnaires using Google Forms after the lesson. The data has been processed quantitatively also using Google Forms. The conclusions follow the results at the end of the chapter.

The questionnaire

General questions
Name and surname:
School:
Grade:
Country:
Lesson:

- Bicycle factory
- Braking distance
- Spreading the rumours
- Slide
- Job advertisement
- Conflict lines - introduction
- Conflict lines - parabola
- ab-ba
- Achilles and the turtle
- Doubling trig formula
- Geometric series

Lesson related questions

1. Did you have any difficulties understanding the goals of the scenario?
   Mark on a scale from 1 to 5.
   no difficulties at all — a lot of difficulties
   1 2 3 4 5
   If yes, describe them.

2. Were Prerequisite mathematical knowledge, Grade, Time and Required material appropriately planned?
   Mark on a scale from 1 to 5.
   not appropriately planned — excellent planning
   1 2 3 4 5
   If not, what were your observations?

3. Was the time allotted for the task implementation in each phase well planned?
   Mark on a scale from 1 to 5.
   not appropriately planned excellent planning
   1 2 3 4 5
   If not, what were your observations?

4. Which tasks (Teacher's or Students' actions) were incomplete / unsuitable?

5. Would you suggest any additional ideas for section "Possible ways for students to realize target knowledge"?

6. Did you notice any mistakes in the scenario? If yes, what were they?

7. Did you change anything in the scenario? If yes, what? Did you use any additional materials?

8. Evaluate the effort of your students - did your students work harder than usually in this class?
   1 - Much less
   2 - Less
   3 - The same
   4 - More
   5 - Much more

9. Which students' reactions/answers showed you that they have completed a certain phase successfully?

10. Please describe your observations from practice: Describe students' actions in each of the devolution. Were there any "unsuccessful" strategies? Which strategies did students use most often? Was there a strategy that nobody used? Did students misunderstand anything?

11. At the end of the lesson (or soon after), did you use any follow up tasks for a quick test of the knowledge the students have developed during the scenario. If yes, please state them. If not, do you have any proposals for such tasks?

12. If you have any suggestions for further problems regarding the topic of the scenario, please state them.

13. Do you have any additional comments? What didn't you like?

Selected results and interpretation

There were 19 responses in the first round for the scenario Area enlargement: 8 from Croatia, 8 from Slovenia, 2 from Denmark and 1 from the Netherlands. These responses were used to refine the questionnaire, to prepare the project team for the workshop discussion with the teachers and to write the MERIA template for scenarios and modules. In the end, materials and responses from the teachers were used to prepare the MERIA modules and their comments were incorporated

in the section describing the possible variations and the didactical variables. Examples of teachers' suggestions are to reduce/increase the time in a certain phase, to change the motivational examples, to use ICT etc.

In the second round, there were altogether 51 responses from 31 teachers (16 responses in Croatia, 20 in Slovenia, 13 in the Netherlands and 2 in Denmark). As the answers are sometimes long and repetitive, we only provide a digested version of selected responses.

Teachers were introduced to scenarios by the project team and in almost all cases they had no difficulty to understand the main goals of the scenario. We had a few very valuable remarks and discussions which led to improvements of the scenarios Slide and Spreading the rumour.

According to answers to Question 2 the teachers find the scenarios well prepared (average grade above 4 out of 5). Issues faced are related to the time needed to prepare implementation of a scenario designed by someone else or to implement a scenario with a topic that is not being covered at the moment (not fitting with the flow of the yearly plan). Time frame is the most discussed didactical variable. We have tried to analyse and incorporate teachers' complaints in the subsequent versions of the scenarios.

Almost all tasks were suitable in the classes in which the scenarios were implemented, but we were convinced that the scenarios Conflict lines – Parabola, Achilles and the turtle and the Trigonometry doubling formula were much more abstract than the others and too hard for some groups of students. Answers to Question 5 were short, usually with no suggestion from the teachers for further related tasks and problems. This might also indicate that teachers are more interested in the implementation than the analysis or task design. Also, in Question 11 teachers say that they did not have time for evaluation tasks, and some of them gave them as homework assignments or did them in the next lesson. There was no reflexion on the quality of the tasks.

The teachers in Croatia and Slovenia report that their students were more engaged than during the regular classes, and in the Netherlands the engagement was reported at the same level. This is in line with the answers of students on their perception how hard they worked.

In answers to Questions 9 and 10, teachers mostly describe what the students were doing in each phase. Their main cue for students' understanding seems to be the "aha!" response. Some teachers report that it is hard for them to answer these questions. This was also discussed with the teachers in person. The impression of the project team was also that teachers find it much easier to discuss students' work than their own actions. On the other hand, reflecting on these questions is an important part of professional development for the teachers changing their style of teaching. Overall, teachers show satisfaction with the lessons, in particular for the five scenarios we have in the end chosen to publish.

Evaluation of MERIA scenarios – students' perspective

Prepared and analysed by: Kristijan Cafuta and Selena Praprotnik, University of Ljubljana, Slovenia; Matija Bašić, Faculty of Science, University of Zagreb, Croatia; Sanja Antoliš, XV. Gymnasium, Zagreb Croatia

MERIA scenarios were tested in class. In the first round, answers were gathered after the implementation of the pilot scenario Area enlargement in all four countries. This facilitated the project team to develop guidelines for development of other scenarios. In the second round, there were 11 different scenarios in total and we used the results together with feedback from the teachers during workshops to decide which five scenarios to finalize. In both rounds, students completed questionnaires using Google Forms after the lesson. The data has been processed quantitatively also using Google Forms. The conclusions follow the results at the end of the chapter.

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- Achilles and the turtle
- Doubling trig formula
- Geometric series

Lesson related questions

1. What are the biggest differences in today's lesson and the traditional lessons in your opinion?

2. Did you find the instructions for the lesson clear and understandable?
   Mark on a scale from 1 to 5.
   not understandable at all — completely understandable
   1 2 3 4 5

3. Compare the lesson to a traditional one.
   1 - Much less interesting
   2 - Less interesting
   3 - The same
   4 - More interesting
   5 - Much more interesting

4. Was there enough time to do each prescribed task?
   1 - Not at all
   2 - A little more would be OK
   3 - Enough
   4 - A little too much
   5 - More than enough

5. Did you encounter any difficulties in understanding and solving the tasks?
   Mark on a scale from 1 to 5.

no difficulties at all				a lot of difficulties
1	2	3	4	5

6. Do you find mathematics more meaningful (related to your life) because of this lesson?
   1 - Yes
   2 - No
   3 - Do not know

7. How often would you like to have lessons that implement today's teaching method?
   1 - Never
   2 - Less than once per month
   3 - Once per month
   4 - Every fortnight
   5 - Every week

8. How would you rate today's lesson?
   Mark on a scale from 1 to 5.

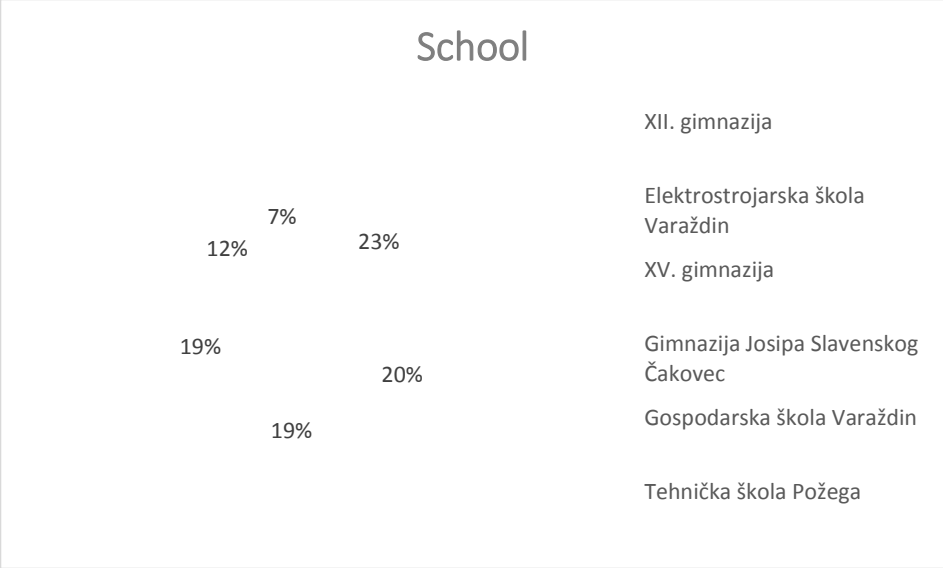
very bad				excellent
1	2	3	4	5

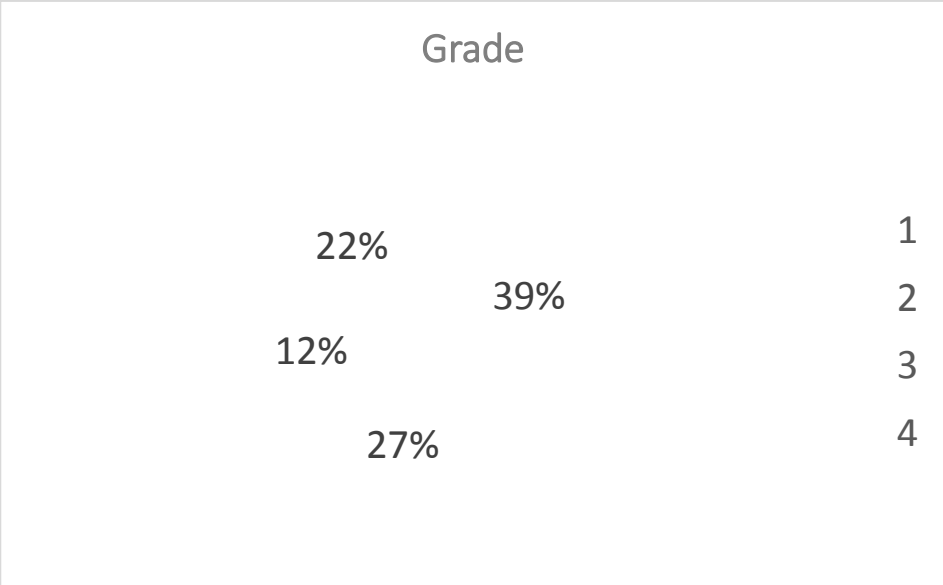
9. Please, try to describe the lesson using (at least) one word.

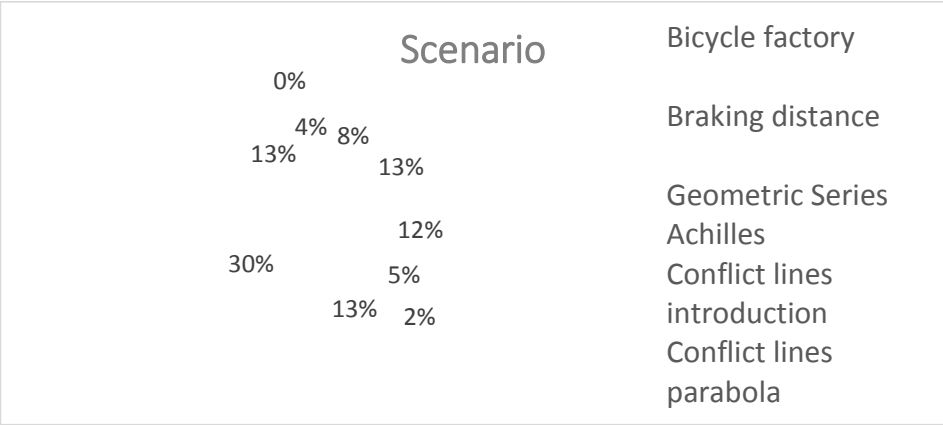
10. Evaluate your effort - did you work harder than usually in regular lessons?
    1 - Much less
    2 - Less
    3 - The same
    4 - More
    5 - Much more

Results of the second round, by country

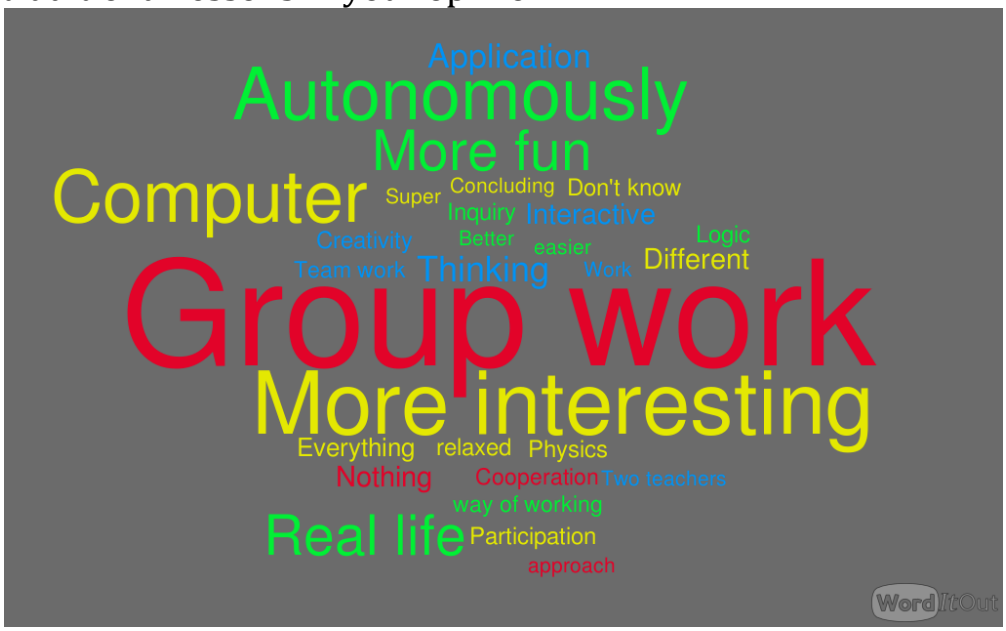
Croatia







1. What are the biggest differences in today's lesson and the traditional lessons in your opinion?



2. Did you find the instructions for the lesson clear and understandable?

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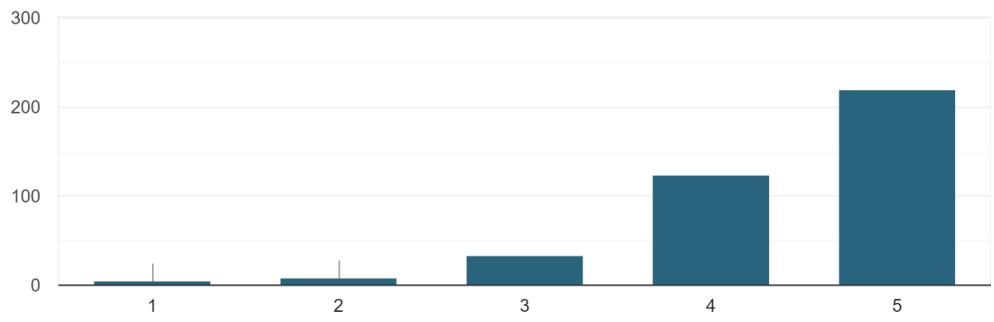

3. Compare the lesson to a traditional one.

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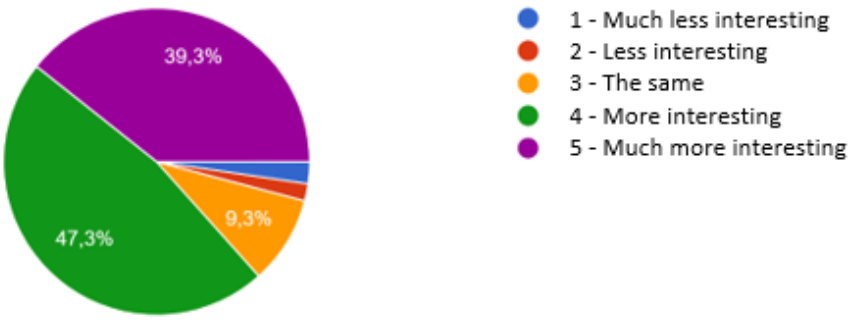
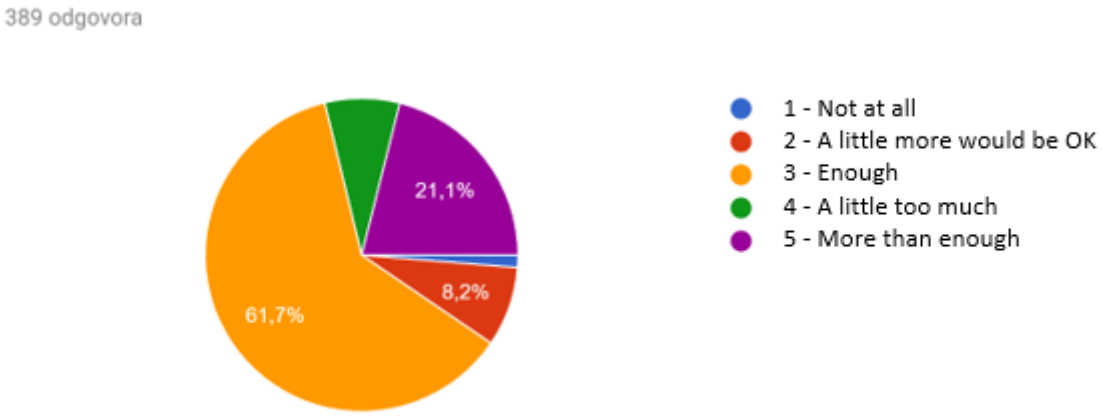
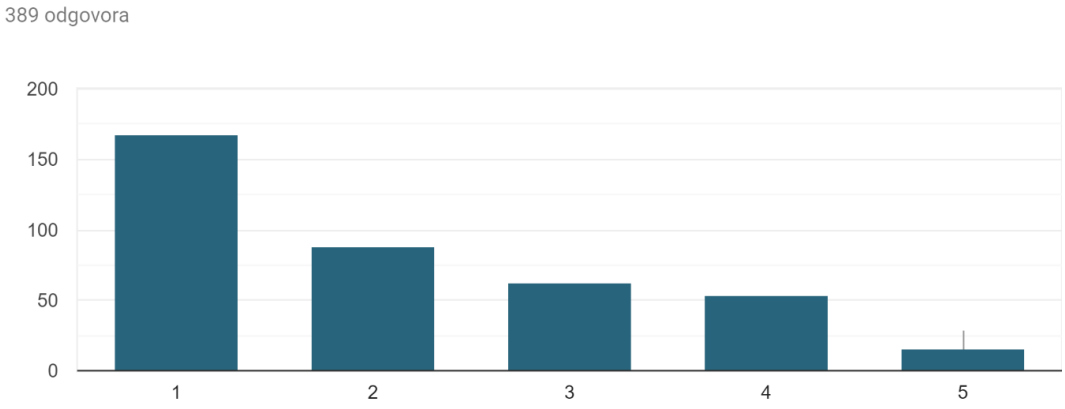


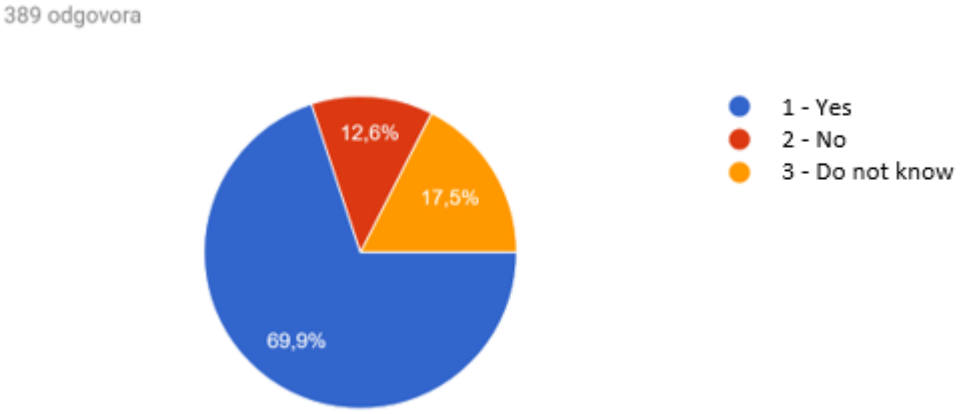
4. Was there enough time to do each prescribed task?



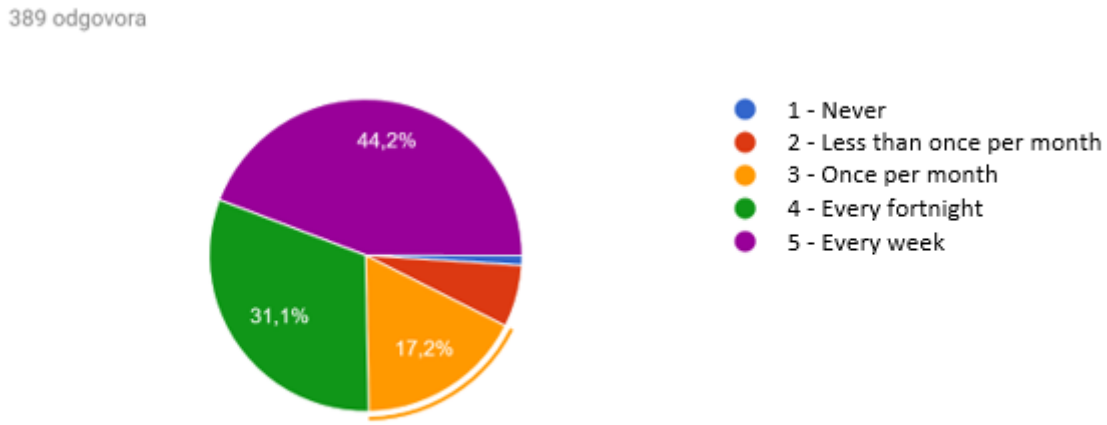
5. Did you encounter any difficulties in understanding and solving the tasks?



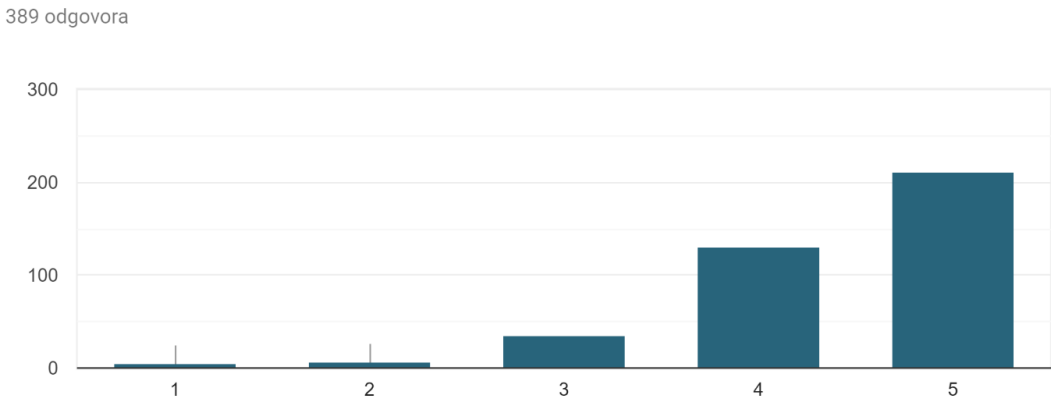
6. Do you find mathematics more meaningful (related to your life) because of this lesson?



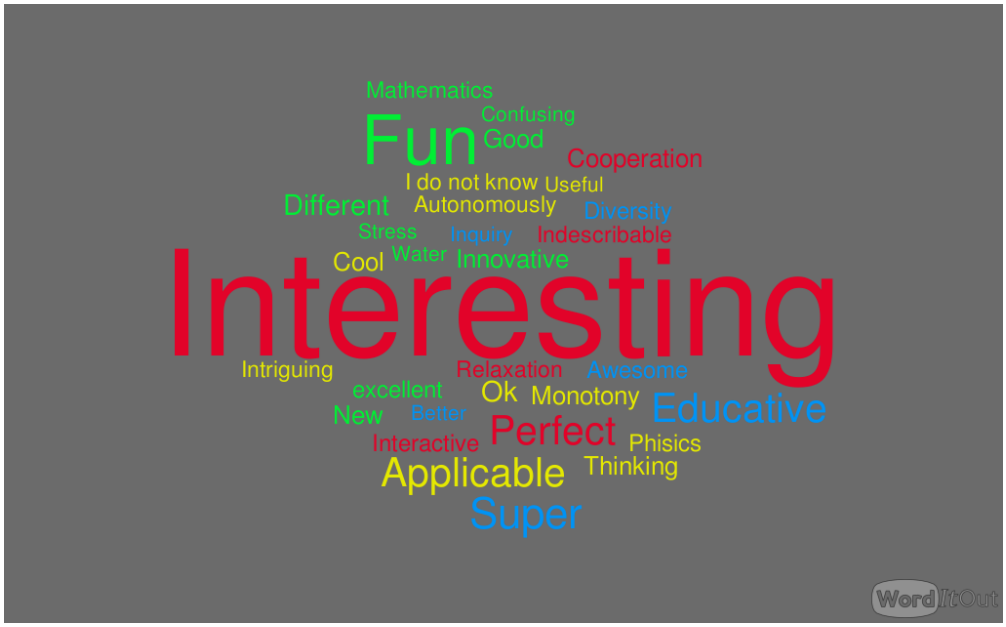
7. How often would you like to have lessons that implement today's teaching method?



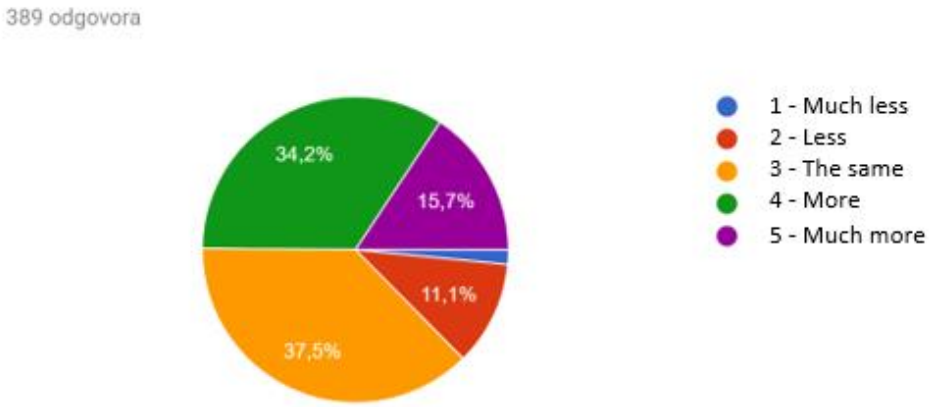
8. How would you rate today's lesson?



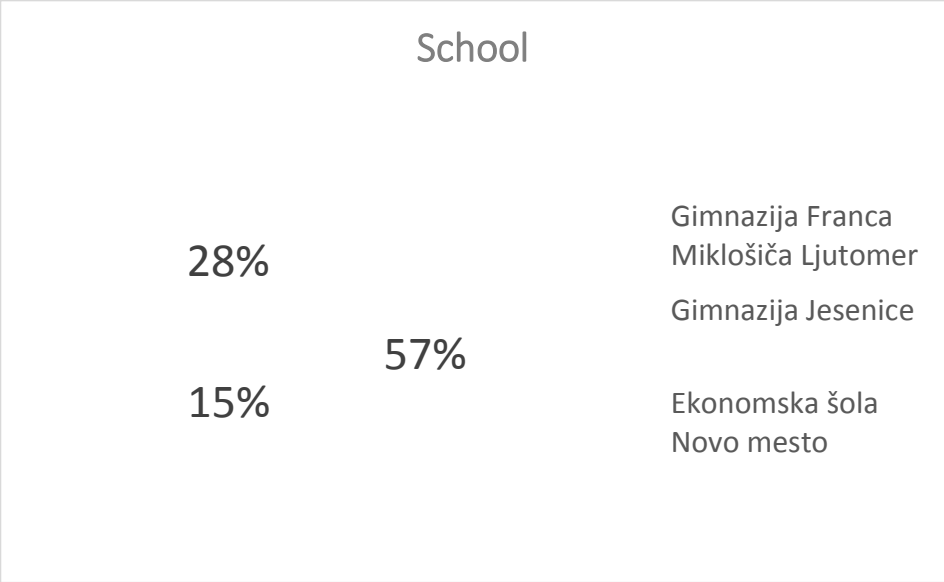
9. Please, try to describe the lesson using (at least) one word.



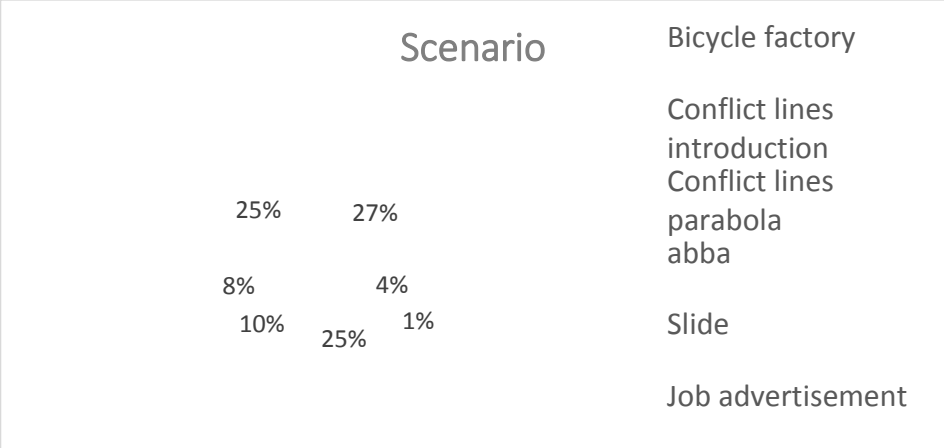
10. Rate your efforts - are you doing more than in ordinary classes in mathematics?



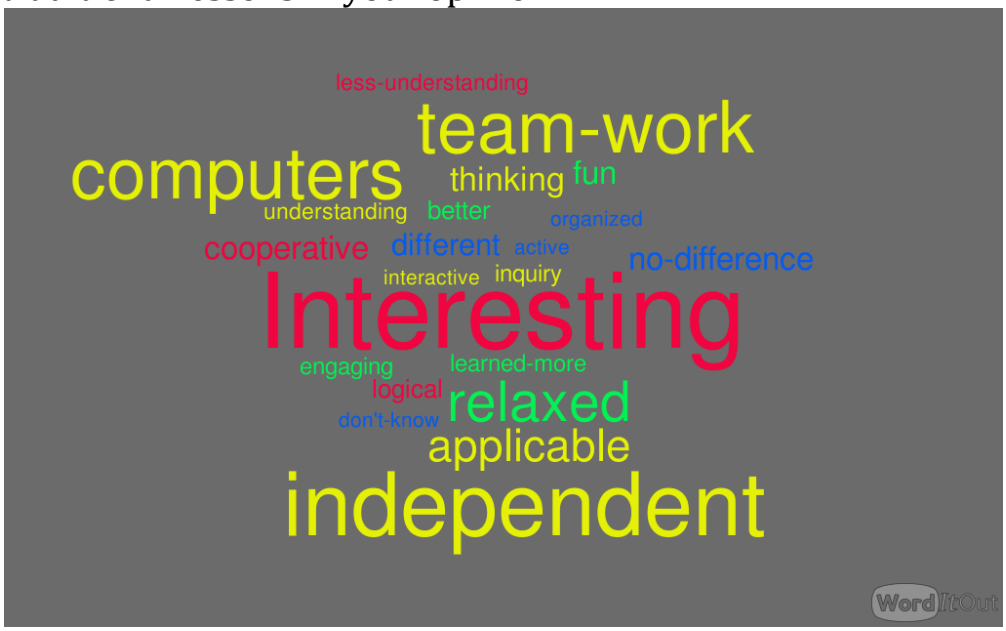
Slovenia


School
28%
57%
15%
Gimnazija Franca Miklošiča Ljutomer
Gimnazija Jesenice
Ekonomska šola Novo mesto

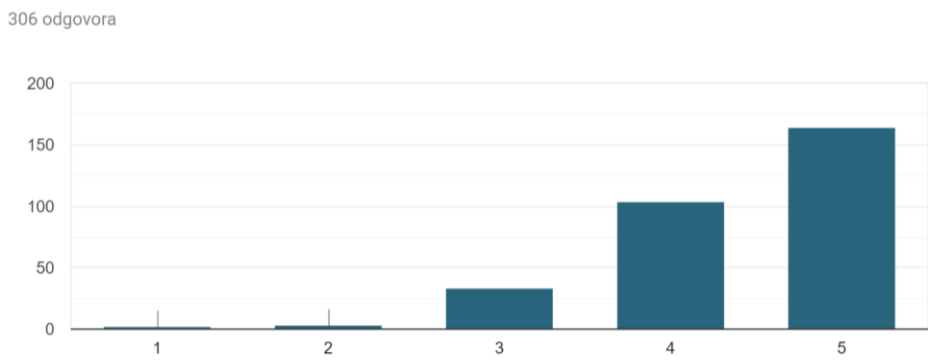
Grade
16%
27%
30%
27%
1
2
3
4


Scenario
25%
27%
8%
4%
10%
25%
1%
Bicycle factory
Conflict lines introduction
Conflict lines parabola abba
Slide
Job advertisement

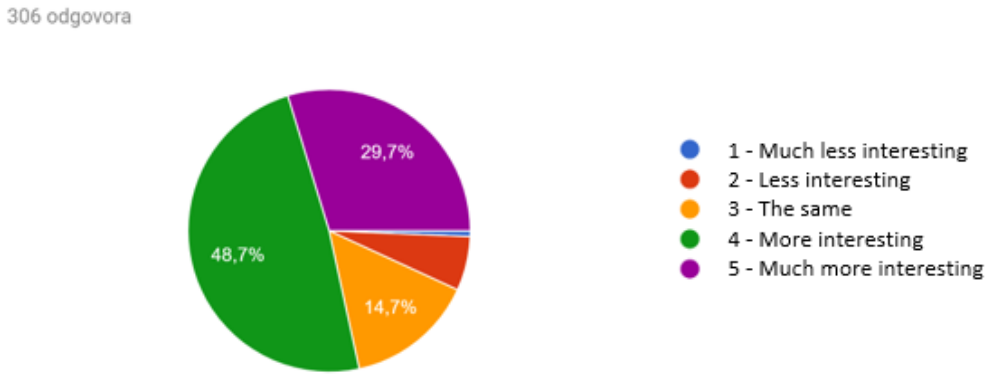
1. What are the biggest differences in today's lesson and the traditional lessons in your opinion?



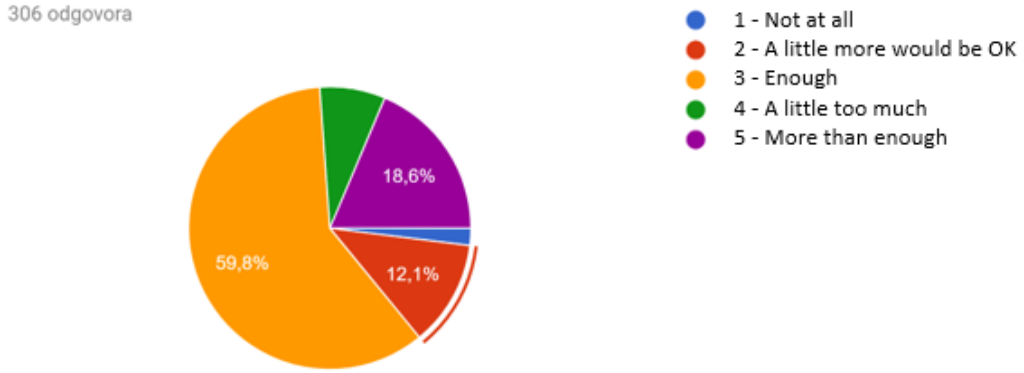
2. Did you find the instructions for the lesson clear and understandable?



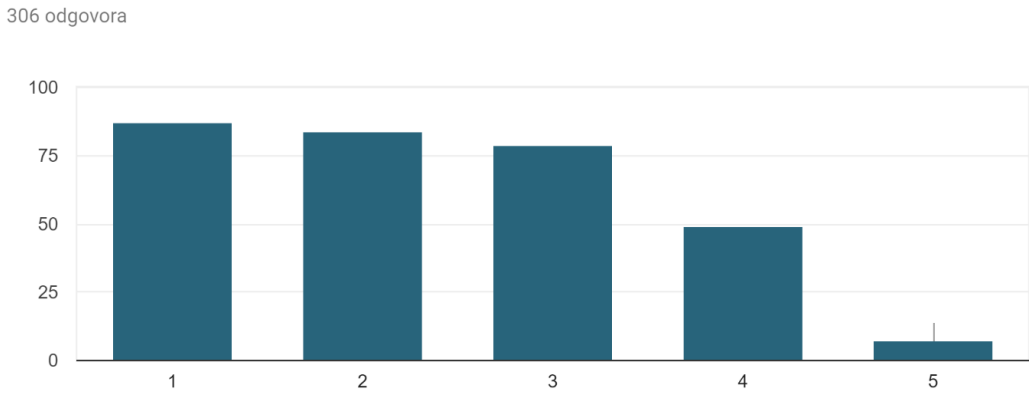
3. Compare the lesson to a traditional one.



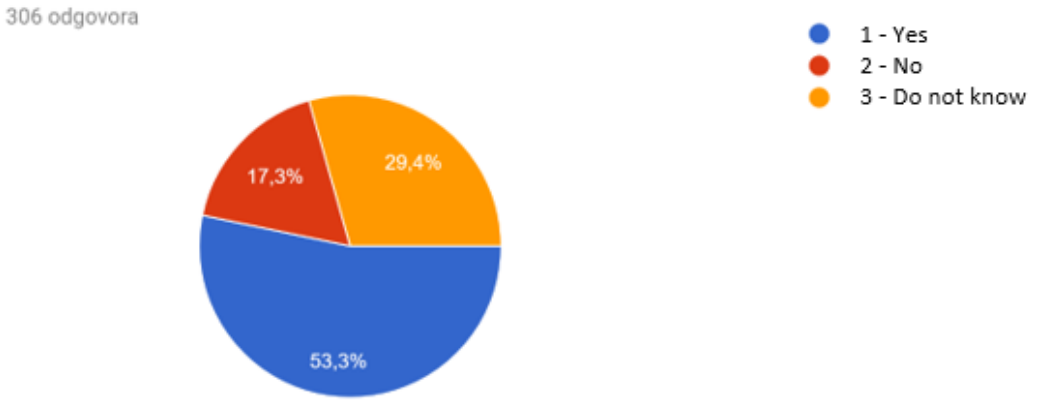
4. Was there enough time to do each prescribed task?



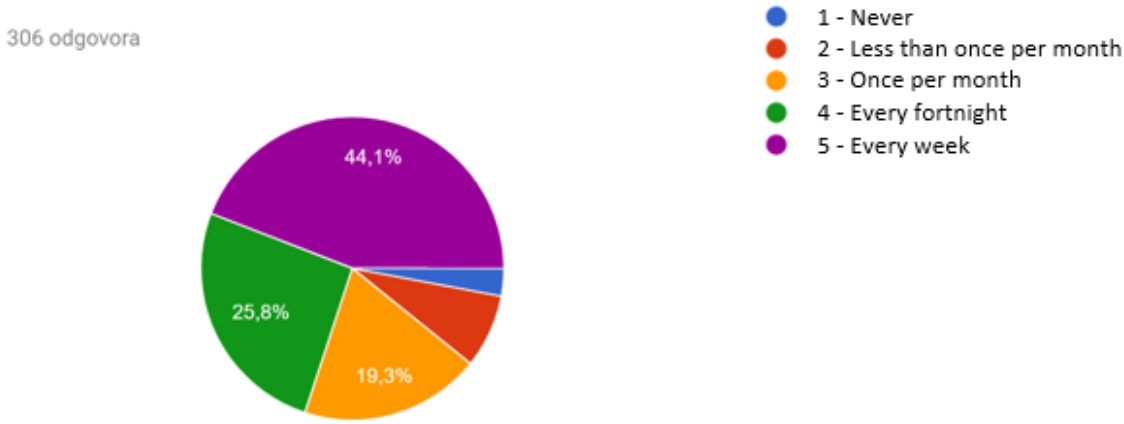
5. Did you encounter any difficulties in understanding and solving the tasks?



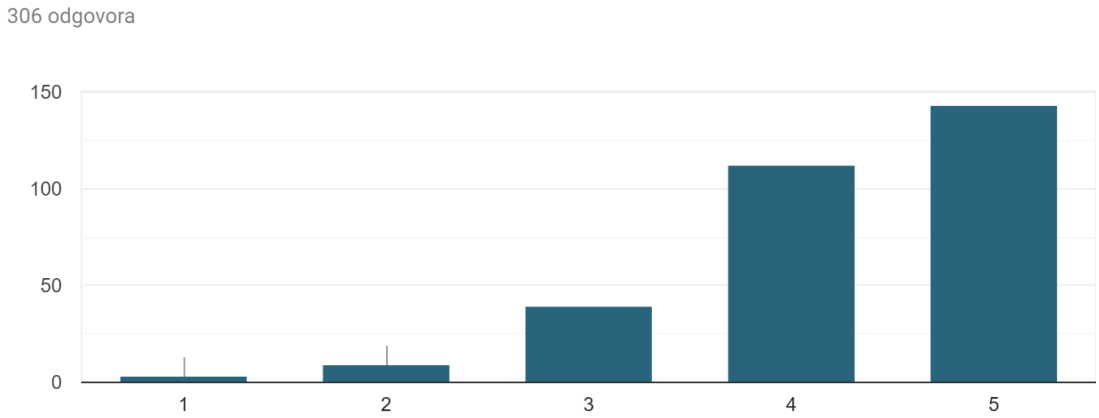
6. Do you find mathematics more meaningful (related to your life) because of this lesson?



7. How often would you like to have lessons that implement today's teaching method?

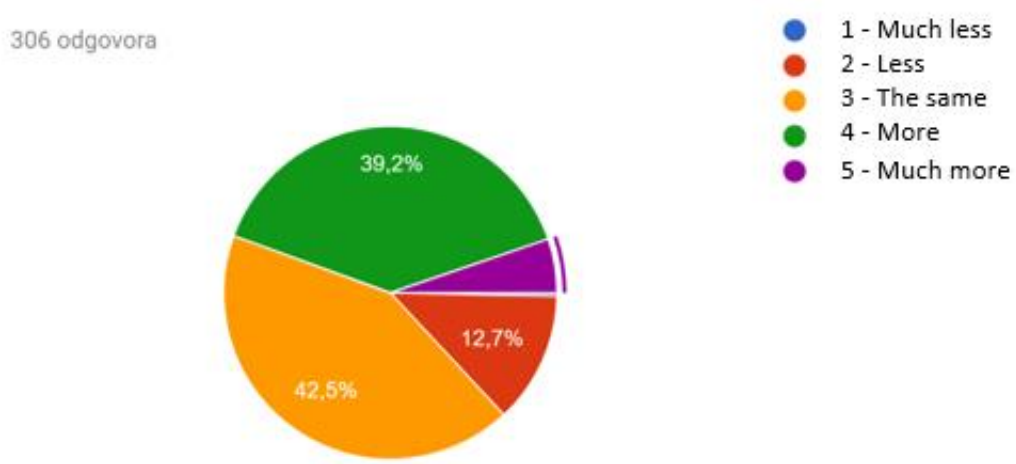


8. How would you rate today's lesson?

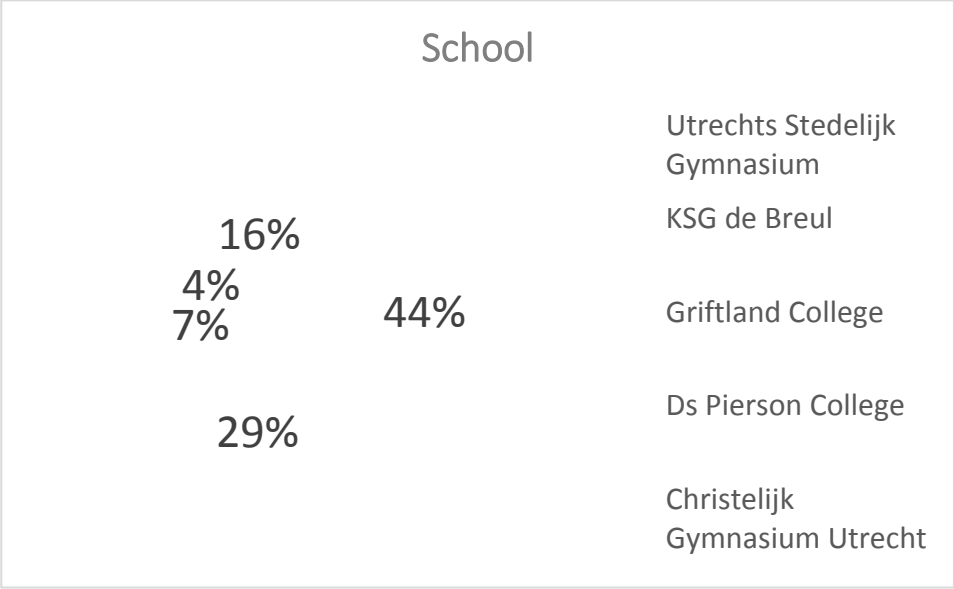


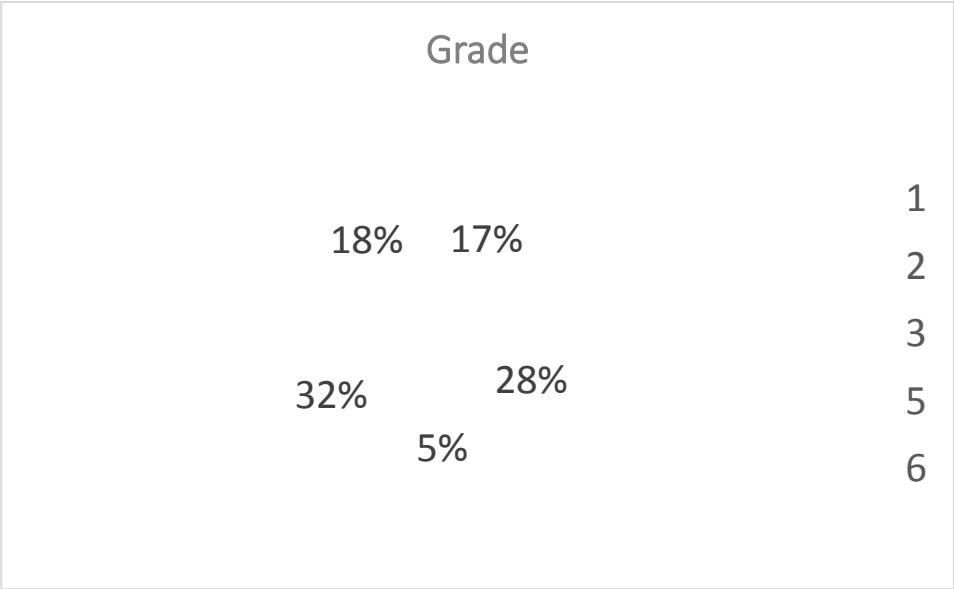
9. Please, try to describe the lesson using (at least) one word.

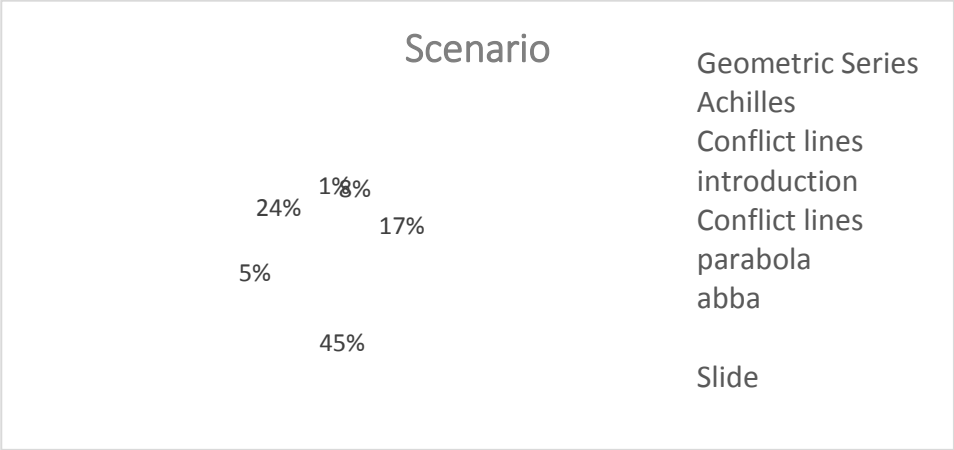
10. Rate your efforts - are you doing more than in ordinary classes in mathematics?



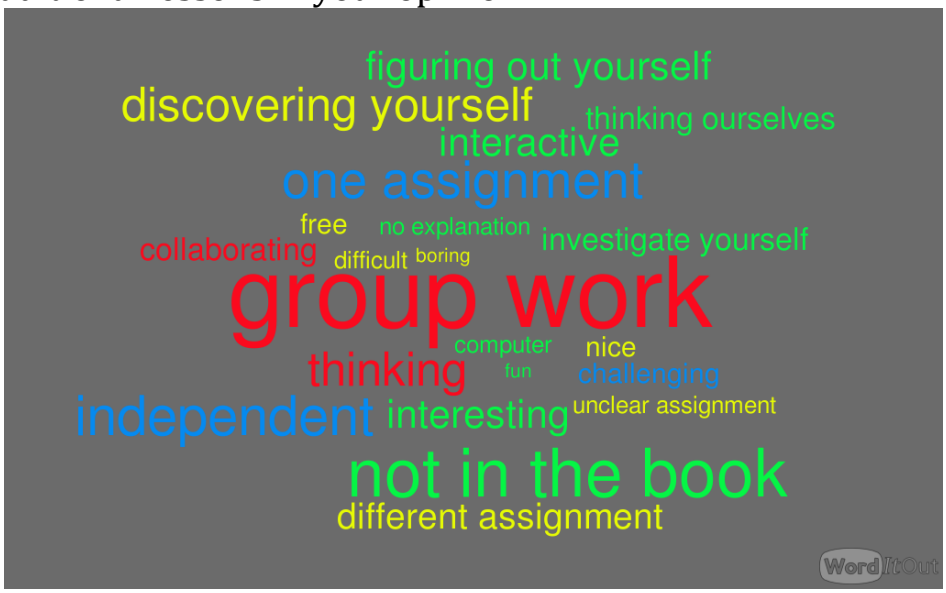
The Netherlands







1. What are the biggest differences in today's lesson and the traditional lessons in your opinion?



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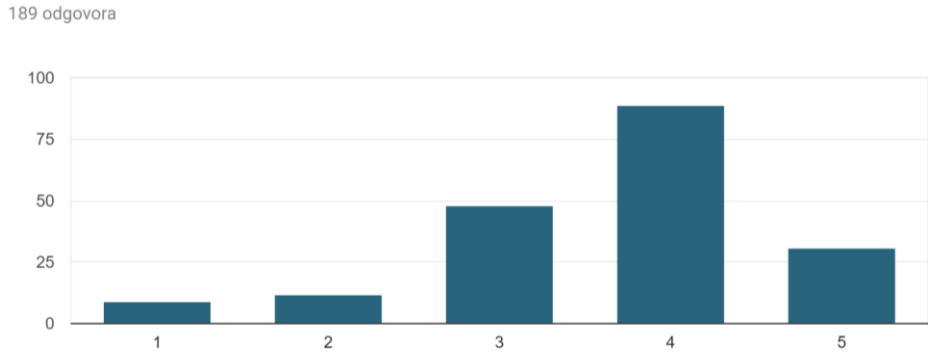
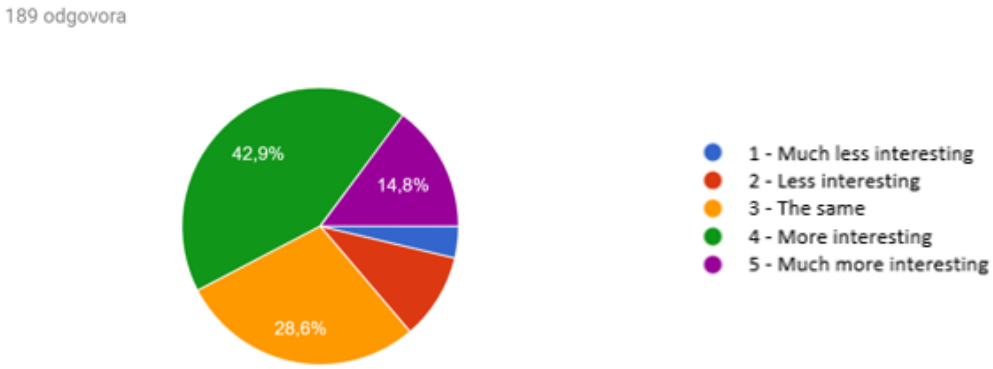


3. Compare the lesson to a traditional one.

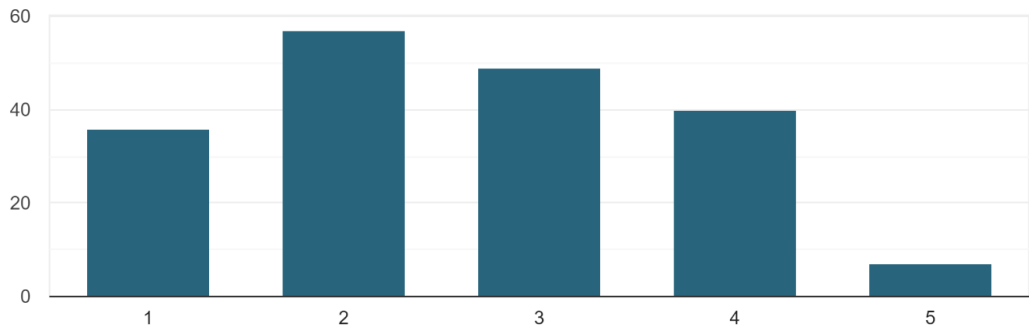


4. Was there enough time to do each prescribed task?

189 odgovora

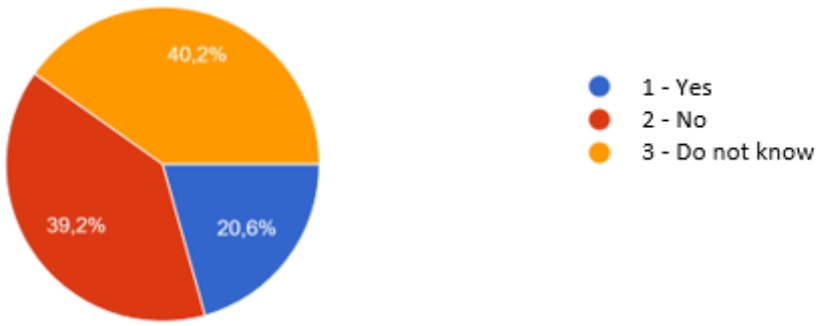
5. Did you encounter any difficulties in understanding and solving the tasks?

189 odgovora



6. Do you find mathematics more meaningful (related to your life) because of this lesson?

189 odgovora

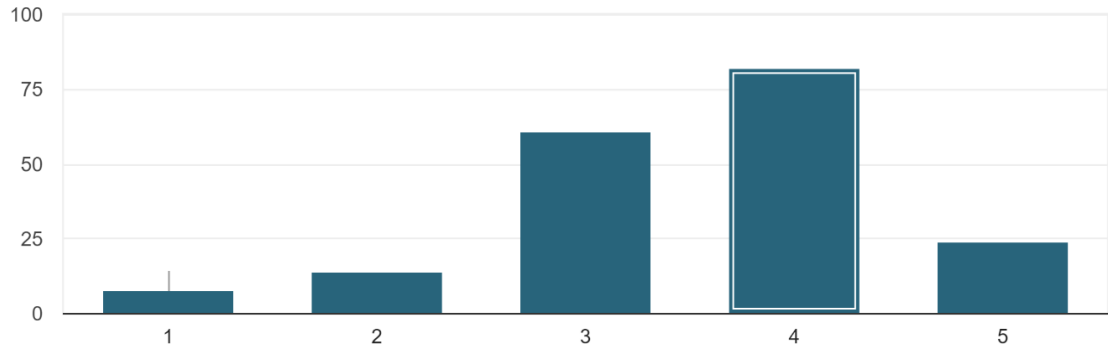


7. How often would you like to have lessons that implement today's teaching method?

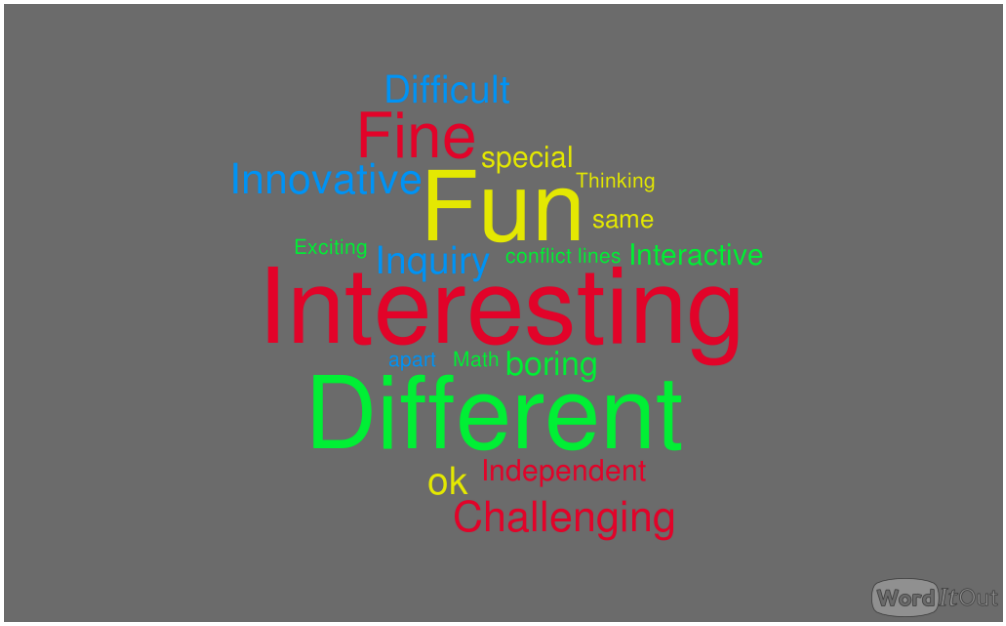
189 odgovora

8. How would you rate today's lesson?

189 odgovora

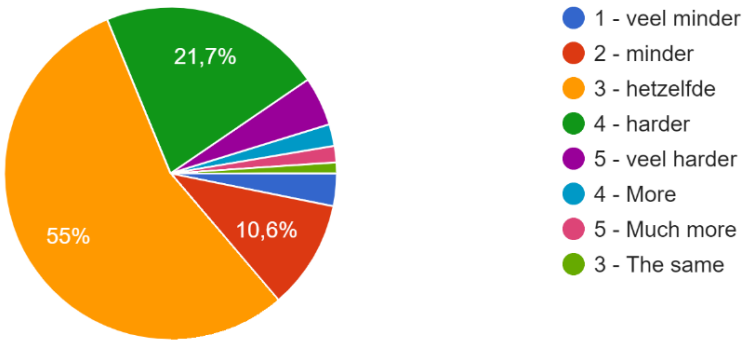


9. Please, try to describe the lesson using (at least) one word.

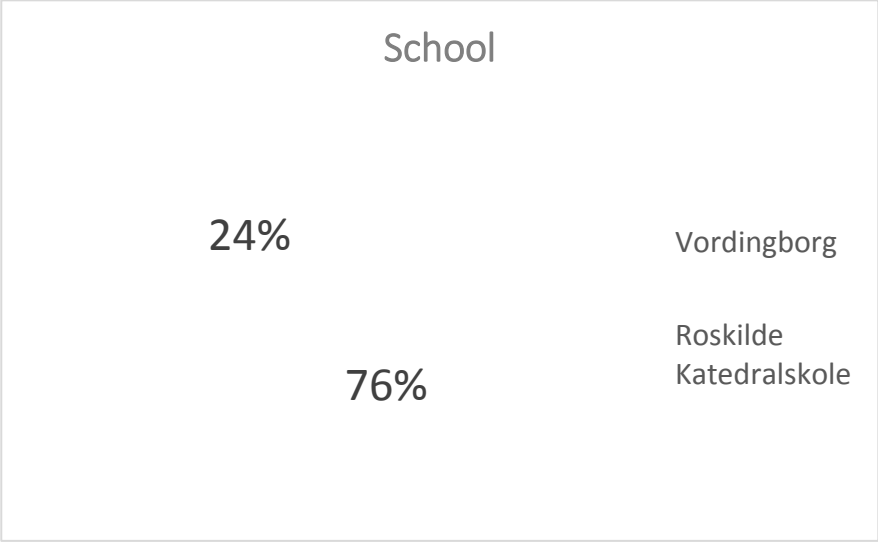


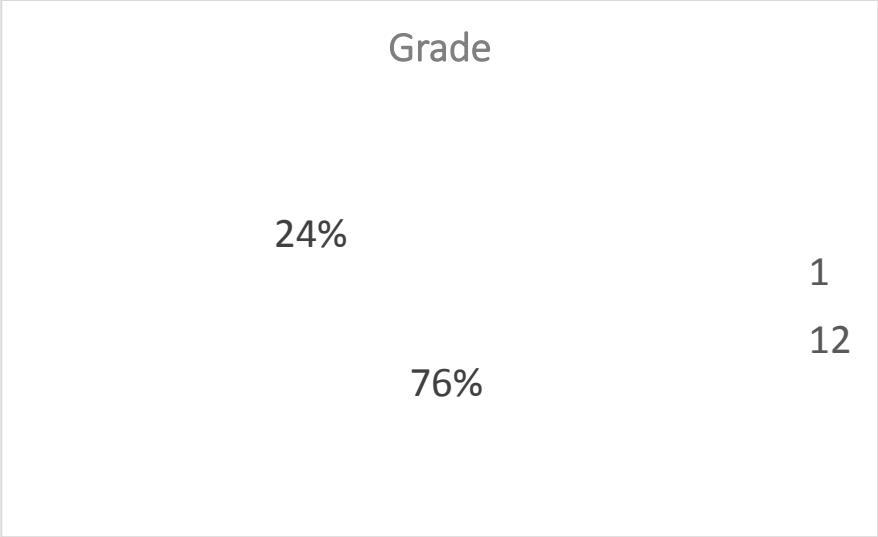
10. Rate your efforts - are you doing more than in ordinary classes in mathematics?

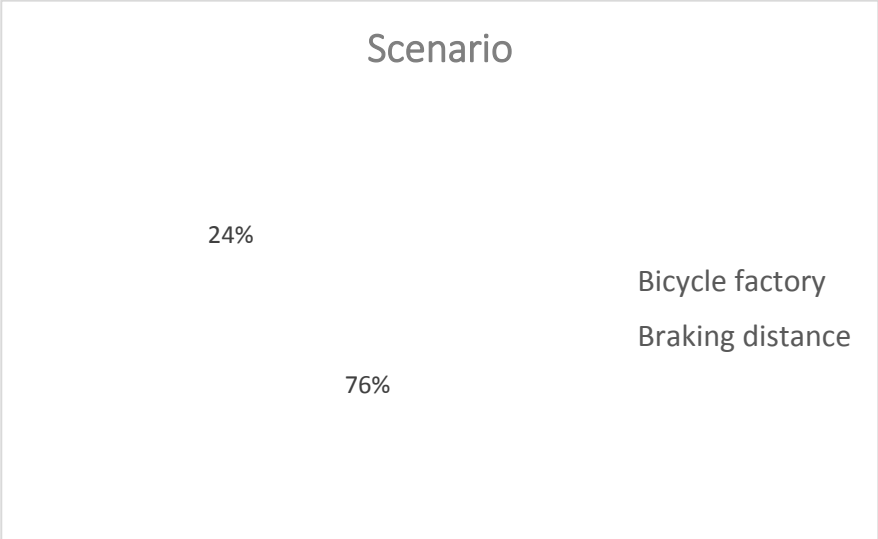
189 odgovora



Denmark

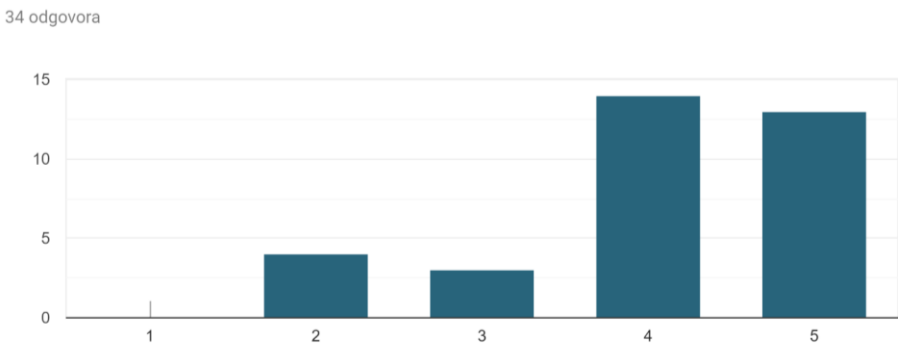

School
24%
76%
Vordingborg
Roskilde Katedralskole


Grade
24%
76%
1
12

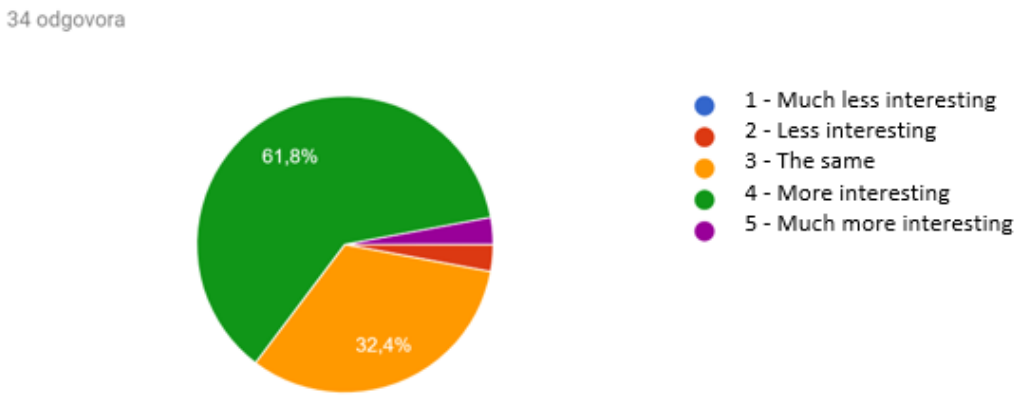

Scenario
24%
76%
Bicycle factory
Braking distance

1. What are the biggest differences in today's lesson and the traditional lessons in your opinion?

2. Did you find the instructions for the lesson clear and understandable?

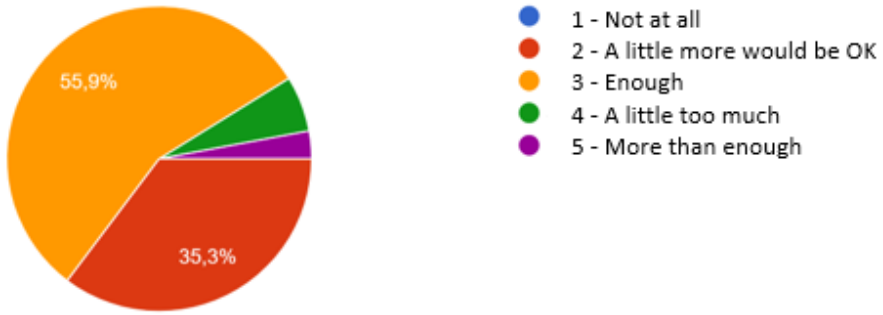


3. Compare the lesson to a traditional one.



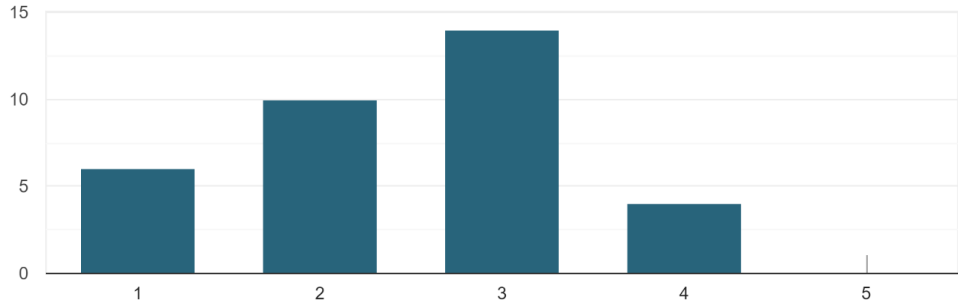
4. Was there enough time to do each prescribed task?

34 odgovora



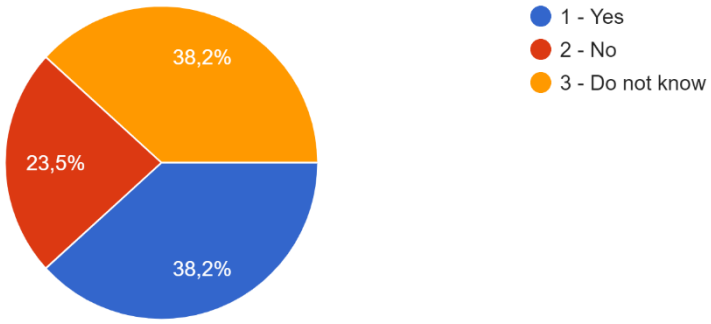
5. Did you encounter any difficulties in understanding and solving the tasks?

34 odgovora



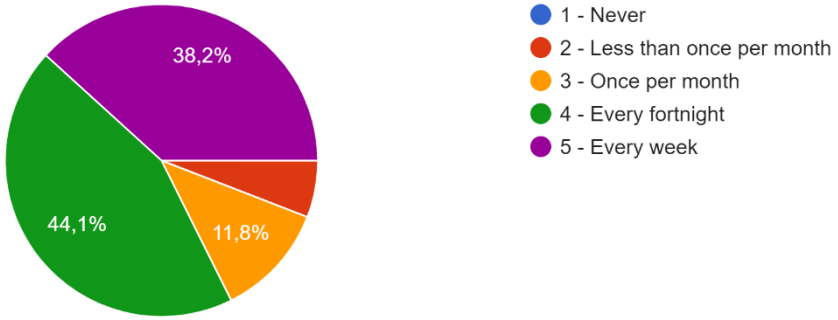
6. Do you find mathematics more meaningful (related to your life) because of this lesson?

34 odgovora



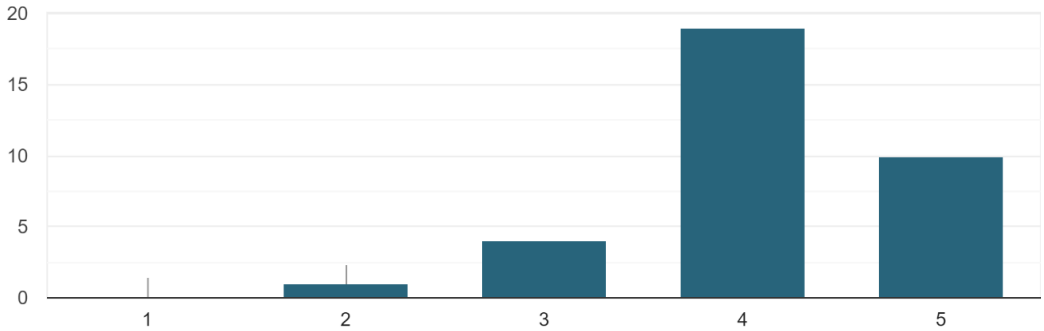
7. How often would you like to have lessons that implement today's teaching method?

34 odgovora



8. How would you rate today's lesson?

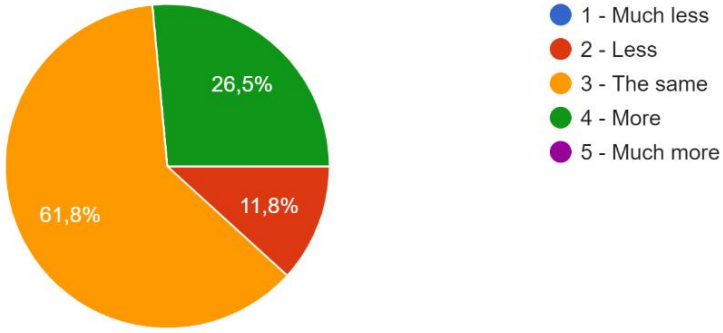
34 odgovora



9. Please, try to describe the lesson using (at least) one word.

10. Rate your efforts - are you doing more than in ordinary classes in mathematics?

34 odgovora



Conclusion

With this questionnaire a lot of data has been gathered on a reasonably big sample of student population, but it must be emphasized that the research was much more oriented on probing students' reflexion on the lesson to understand their interests than to validate the methodology behind the design of the scenarios. Hence, we not claim any significance of the conclusions that we present, but we explain which data we considered most important. We should note that the sample is considerably smaller in Denmark (34 students) as opposed to other three countries (189 in the Netherlands, 306 in Slovenia and 389 in Croatia).

Questions 2, 4, 5 and 8 show that the students find MERIA scenarios clear, well timed, not too difficult to engage with and they grade the lesson very positively.

Quite clearly, the answers indicate that the students find MERIA scenarios *interesting*. In Question 3, a majority of students explicitly say that the lesson was more interesting than a traditional one. The number of students claiming this is extremely high in Croatia (90%) and Slovenia (78%), while above half in Denmark (62%) and the Netherlands (58%). It is interesting to note that the lesson was graded less interesting and at the same time slightly less clear in the Netherlands and Denmark. Unfortunately, we could not inquire deeper into the relation of these two characteristics or control other factors related to the level of preparation or enthusiasm of the teacher.

As to *relevancy* (meaningfulness) of the problems, the situation is less convincing. In Question 6, in Croatia 70% of students answer that the problem is meaningful to them, but only 20% of the students in the Netherlands express in that way.

The number of students that say that they would like to have this type of lessons (Question 7) every week or once in two weeks in very high in Croatia (75%), Slovenia (70%) and Denmark (82%), while these two answers were chosen by 44% of the tested students in the Netherlands. The students mostly report that their level of engagement (Question 10) was the same or higher than compared to the regular lesson – less than 15% of students answered that they worked less.

As each of these characteristics was evaluated based on one question, we remind ourselves also that we have no way to know to what extent the significance of the results is reliable and did the students want to give answers expected by the teacher.

It might be still of interest to analyse the words that the students used to describe the lesson (Questions 1 and 9). In Croatia, Slovenia and the Netherlands the most used words indicate that the students found the lessons *interesting* and *fun*, that they have recognized *working in groups* and *independently* (of the teacher). In Croatia and Slovenia, students noted using *computers*, while in Denmark students noticed the rich character of the problems ("compare ideas", "different angles", "open task"). In general, there is an impression that the students enjoyed the lesson as it brought a fresh new way of teaching with the roles of the students and the teacher changed. Looking into other (not so frequently) mentioned words might also give some confidence that the students recognized the benefits of inquiry based teaching as intended by the project team, e.g. "good content", "confusing", "challenging", "useful", "not in the book", "applicable", "independent thinking", "logical", "cooperation", "participation".

Evaluation of MERIA workshops

Croatia

Prepared by: Sanja Antoliš, XV. Gymnasium, Zagreb, Croatia

The questionnaire was given to the participants of the workshop at the end of the third day.

Questionnaire

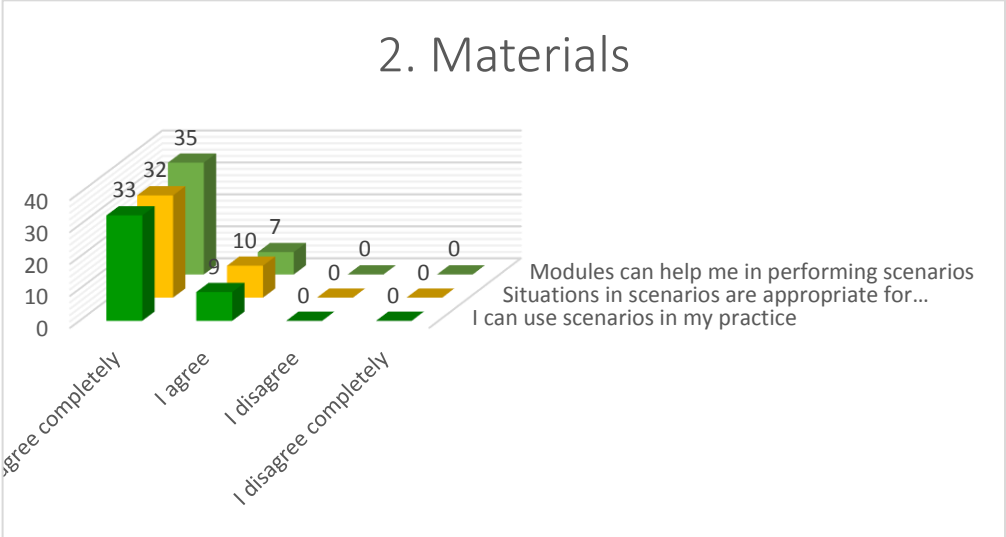
Dear colleague,
We would kindly ask you to answer all the questions because your opinion is very important to us. Please mark to which extent you agree with the statements below.

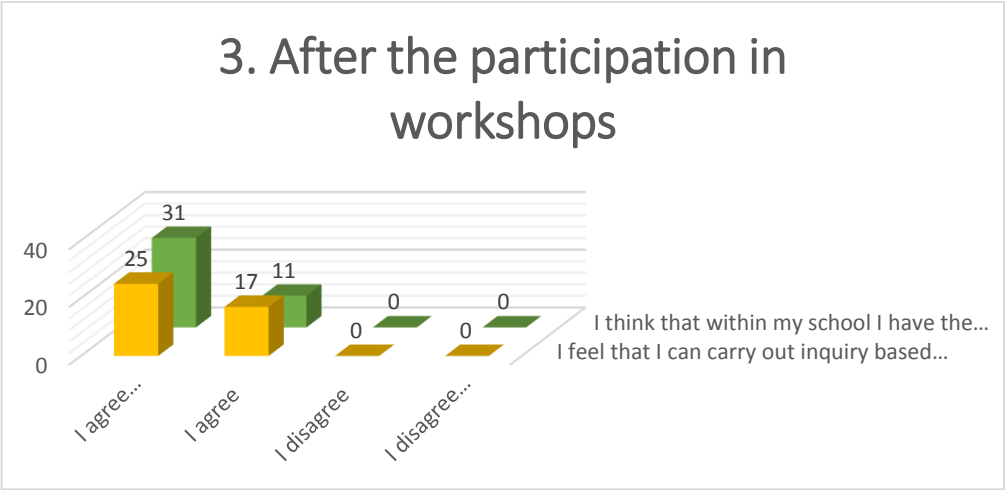
	I agree completely	I agree	I disagree	I disagree completely
1. WORKSHOP				
1.1. In general, it met my expectations				
1.2. It is useful for my professional development				
1.3. It was well organized				
2. MATERIALS				
2.1. I can use scenarios in my practice				
2.2. Situations in scenarios are appropriate for my students				
2.3. Modules can help me in performing scenarios				
3. AFTER THE PARTICIPATION IN WORKSHOPS				
3.1. I feel that I can carry out inquiry based teaching				
3.2. I think that within my school I have the support for continuing to apply the learned methods				

My comment: _____

Results

1. Workshop
40
30
20
10
0
37
37
38
5
5
4
0
0
0
0
0
0
It was well organized
It is useful for my professional development
In general, it met my expectations
gree completely
I agree
I disagree
I disagree completely


2. Materials
40
30
20
10
0
33
32
35
9
10
7
0
0
0
0
0
0
Modules can help me in performing scenarios
Situations in scenarios are appropriate for...
I can use scenarios in my practice
gree completely
I agree
I disagree
I disagree completely


3. After the participation in workshops
40
20
0
25
31
17
11
0
0
0
0
I think that within my school I have the...
I feel that I can carry out inquiry based...
I agree...
I agree
I disagree
I disagree...

Comments

- This is very inspiring, the scenarios, discussions about them… exchange of experiences and ideas is always wow

  ove je jako inspirativno, sami scenariji, razgovori i diskusije o njima ... razmjena inkustava i ideja je uvijek wow...

- Excellent. Thanks. I look forward to continuing ☺

  Odlično. Hvala. Veselim se nastavku ☺

- Everything OK, it was interesting and useful.

  Sve OK, bilo je zanimljivo i korisno.

- Thanks you for new and useful knowledge and experience ☺

  Hvala Vam na novim i korisnim znanjima i iskustvu. :)

- Excellent organization. Interesting and applicable

  Odlično organizirano. Zanimljivo i primjenjivo

- The project is an excellent chance for better communication and exchange of ideas between colleagues from the school, and in particular from the university, I did not have the chance for it before.

  Projekt je izvrsna prilika za bolju komunikaciju i razmjenu ideja među kolegama iz škole, a posebno s fakultetom, što do sad nisam imala prilike.

- Thank you for including me into this project. Very pleasant environment to learn new content.

Hvala na uključivanju u projekt. Vrlo ugodno okruženje učenja novih sadržaja.

- It was interesting and useful!!! It would good to get more scenarios.

Bilo je zanimljivo i korisno!!! Bilo bi dobro imati još takvih scenarija

- One should continue with the project and ideas.

Treba nastaviti s projektom i idejama

- I am satisfied with the implementation of the project. I think you have made something of quality and what other teachers should see in the follow-up workshops, as others would benefit in their work as we have. I would like that you transfer to others the enthusiasm you have transferred to me personally. Thank you for that.

Zadovoljna sam s provedbom projekta. Sigurnom da ste napravili nešto kvalitetno što bi trebali kroz naredne radionice vidjeti i drugi nastavnici te da bi im to koristilo u radu kao što koristi i nama. Željela bih da i ostalima prenesete entuzijazam koji ste prenijeli meni osobno. Hvala vam na tome.

- We should apply this style of teaching more.

Trebalo bi više primjenjivati ovakav vid nastave.

- I am delighted with this approach to teaching. I will certainly implement scenarios with next generations.

Oduševljena sam ovakvim pristupom nastavi. Svakako ću provoditi scenarije i s budućim generacijama

- Excellent structure and organization. This type of workshops with constant discussions and exchange of experience that leads to own reflection can highly help in the quality of working.

Odlično strukturirano i organizirano. Ovakve radionice uz konstantnu raspravu i razmjenu iskustava što navodi na vlastito propitkivanje, uvelike može pomoći u kvaliteti rada.

- Excellent organization, pleasant and useful way to spend time, very well structured.

Vrhunska organizacija, ugodno i korisno potrošeno vrijeme, jako dobro strukturirano

- ☺

- Great ideas, it would be nice if you had more financial support for this type of workshops.

Super ideje, bilo bi lijepo kada bi imali financijsku podršku za ovakve radionice i dalje

- More of this type of workshops, where teaching experience is shared and didactical competencies are strengthened

ŠTO VIŠE OVAKVIH RADIONICA KOJE KAZMJENJUJU NASTAVNIČKA I SKUSTVA I DAČAJU NJIHOVE METOD. KOMPETENCIJE

- Quality organization. Approachable, well organized, working with heart and great motivation.

Kvalitetna organizacija. Pristupačnost, organiziranost, rad pa srcem i velikom motivacijom.

- Excellent and relaxed atmosphere. I am extremely satisfied with the organization, acquired knowledge and this experience. Kudos. ☺

ODLIČNA I OPUŠTENA ATMOSFERA. IZNIMNO SAM ZADOVOLJNA ORGANIZACIJOM, STEČENIM ZNANJIMA I OVIM ISKUSTVOM. SVAKA ČAST. ☺

- Perhaps in the future to organize workshops of unstructured task design from the tasks we use in regular lessons.

Možda ubuduće da se organizira radionica izrade nestrukturiranih zadataka iz udžbenika koje koristimo u redovnoj nastavi

- In vocational school more motivation and encouragement is needed than anticipated in the scenarios.

U strukovnim školama potrebno je ipak malo veće vođenje i poticanje pitanjima od predviđenog u scenarijima.

Slovenia

Prepared by: Mojca Suban, National Educational Institute, Slovenia

The questionnaire was given to the participants of the workshop at the end of the third day. Some participants gave written answers other via *Slido* application. We have made a synthesis of all the answers or state the typical answer as follows:

1. Describe the usefulness of the content and material presented at the workshop for your pedagogical practice.

The content and the material are useful for my everyday teaching practice because they are well prepared, and they contain different ways of students solving the problem. The inquiry method is efficient since it pushes students to start thinking and learn better. The method is very time consuming and we do not always have time. Some examples are too difficult for our students.

2. How can you facilitate inquiry based teaching and learning at your school?

Teacher 1: I can try to implement it as much as I can since I am the only teacher of math at our school. All I need is enough time and suitable problems. I have gain sufficient support and motivation at the workshop to continue with this work and encourage students to think.

Teacher 2: I will present the content of the workshop to my colleagues and invite them to observe the inquiry lesson. By doing so I hope to motivate them for trying inquiry based MERIA scenarios.

3. How can MERIA support your inquiry-based lessons?

The support in a form of materials, experiences, literature and seminars/workshops is very welcome. We expect wide variety of scenarios, among which we can choose the ones that suits our students best. We would also like to have a set of »basic« scenarios that we can do in one hour or can be adopted for low achievers. We need ideas for realisations and professional support.

4. How can the MERIA project results become permanent in your teaching practice?

Teacher 1: I will plan at least 2-3 MERIA scenarios for the next school year. I will also do shorter inquiry activities because they do not take so much time.

Teacher 2: I will gradually try out other MERIA scenarios with adjustments according to my students. I will try to produce my own MERIA scenario.

5. What is your message to the teacher trainers at the MERIA workshop?

Cary on what you have started. Spread the idea. Do as many scenarios as you can. The workshop has confirmed my beliefs that we need different instruction and different knowledge. Students can do more than I thought, and I will include inquiry activities in my lessons, along with solving open problems. All I need is to make the first step.

Denmark

Prepared by: Jeanette Axelsen, Denmark

Everyone liked to experience the TDS as part of the learning session and followed by presentation of the different ideas behind. The variation between practical exercises in small groups and presentations was also positively evaluated.

About the time spend on working in small groups, the balance between group sessions, plenum discussions and presentations were more mixed. It is probably because we are different in our ways of learning new concepts and mindsets and producing and discussing new ideas.

Things we could have done differently was primarily to have some more time to develop new ideas and having some more materials to bring back to school. The RME perspective was not necessary and comparing two didactical theories unknown for the teachers participating at the workshop was not that optimal.

The scenarios were well received. Most found them easy to work with or they found it took some time to get into the materials but not impossible. In relation to the time teachers have for preparation, some found it a little heavy.

Almost every teacher says the introduction at the 1st workshop was enough to go home and try out scenarios. Using film materials to illustrate how to teach inquiry based were helpful. And they also found the scenarios relevant compared to the Danish curriculum. But here again the attitudes towards the materials were mixed.

Finally, there were some comments about how the workshops and materials had inspired – not perhaps to follow the TDS scenarios in full, but at least to use the ideas of letting the students work more inquiry based. In relation to the new curriculum (2017) the teachers could see the relevance of inquiry based teaching. The very concrete materials were also mentioned as a very positive aspect of the MERIA workshops.

Evaluation of the MERIA Final Conference

Evaluation questionnaire

The questionnaire was given to the participants of the Final Conference at the end of the second day.

Questionnaire

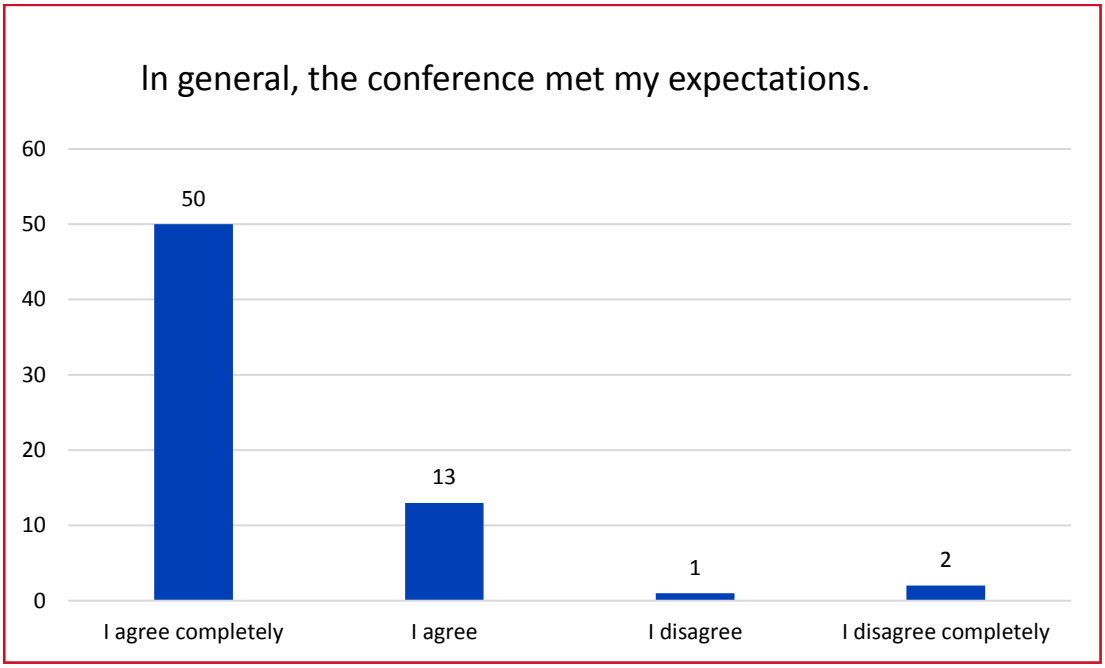
Dear colleague,
We would kindly ask you to answer all the questions because your opinion is very important to us. Please mark to which extent you agree with the statements below.

	I agree completely	I agree	I disagree	I disagree completely
In general, the conference met my expectations.				
The conference was well organized.				
I like the format of the activities.				
I would recommend this conference to my colleague.				
In the future I will try to implement MERIA in my classroom.				

I liked the most…

Suggestions for improvement…

Results


In general, the conference met my expectations.
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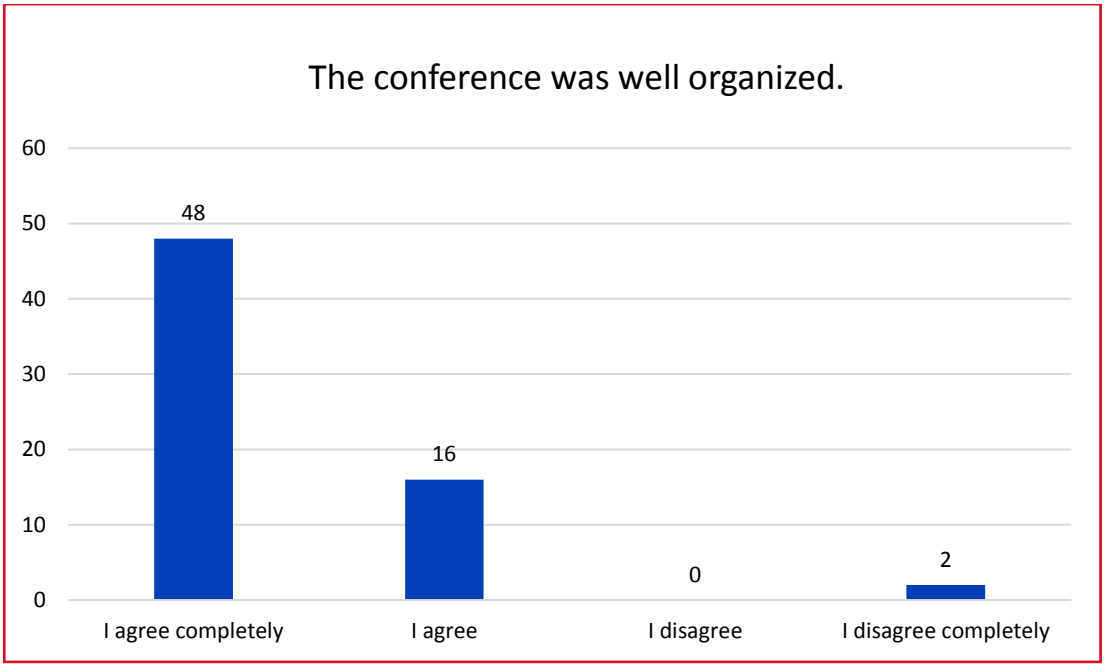
The conference was well organized.
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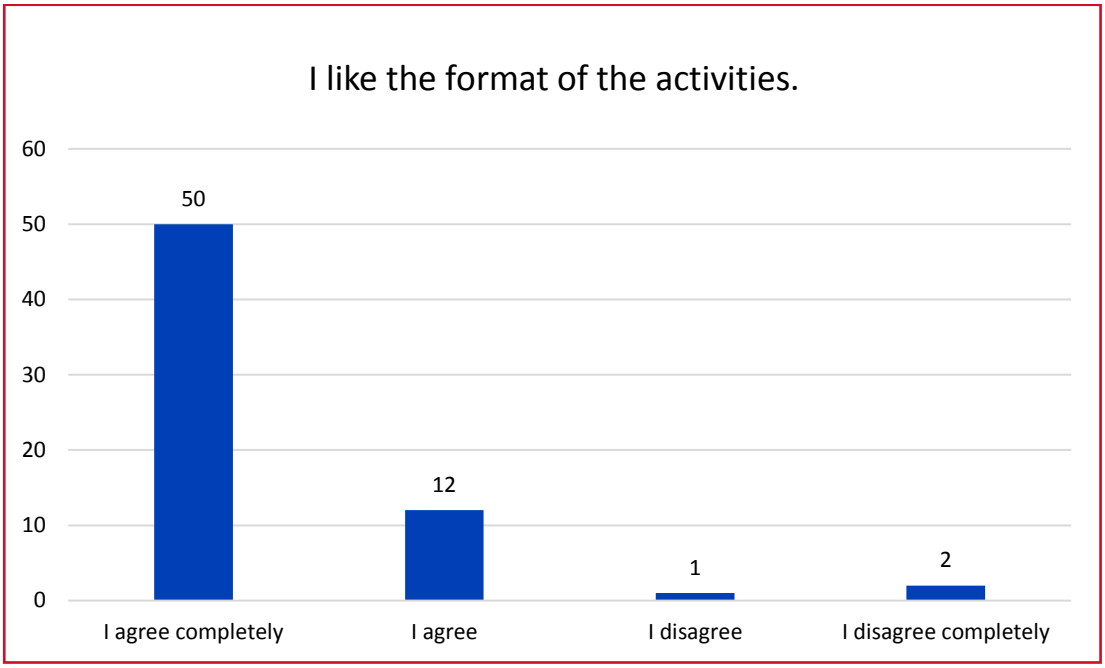
I agree completely
I agree
I disagree
I disagree completely


I like the format of the activities.
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I agree completely
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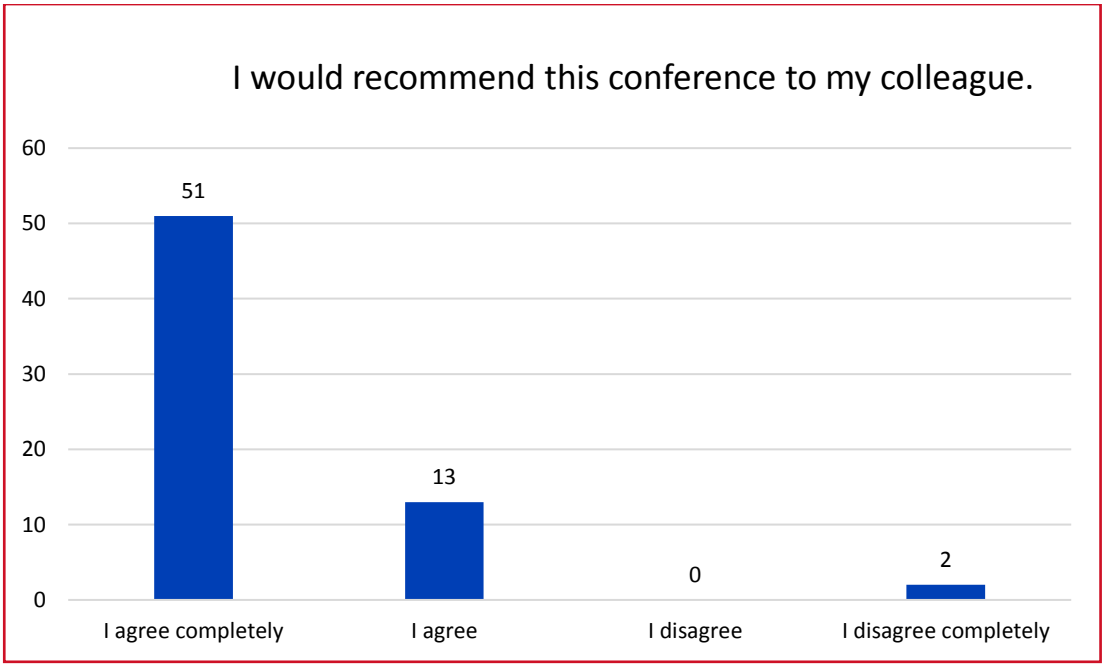
I disagree completely


I would recommend this conference to my colleague.
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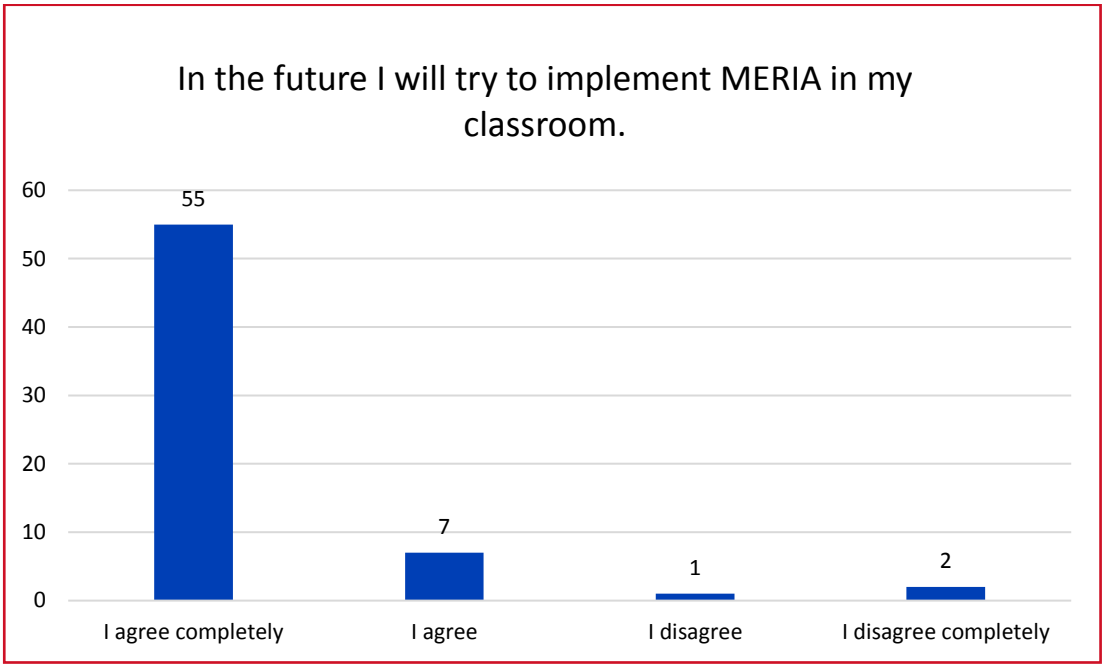
I agree completely
I agree
I disagree
I disagree completely



I liked the most ...

Suggestions for improvement...

- First morning was too formal and had not much content
- More scenarios
- Invite countries that are more used with study lessons
- Workshops in more rooms, it has been very noisy

- Microphone for all
- Hard to see presentations
- Have people sign up for sessions on big paper posted in the entry instead of passing paper around during talks
- The poster session and drinks not to be separated
- Not having lunch in a small classroom
- Break with seeing the city
- More time
- Workshop with teachers of one country

Some comments …

I liked the most... the way of thinking, usefullness and importance of the project. Today we have the opportunity to participate in a lot of the lessons and workshops but for the teacher is important to recognize the tree in the forest of this workshops and online lessons

Suggestions for improvement... Quality before quantity! Thanks!

It was great – thank you for great, great, great event ♡

Suggestions for improvement...

Just stay as you are! With all your enthusiasm to do an improvement on math teaching

Suggestions for improvement...

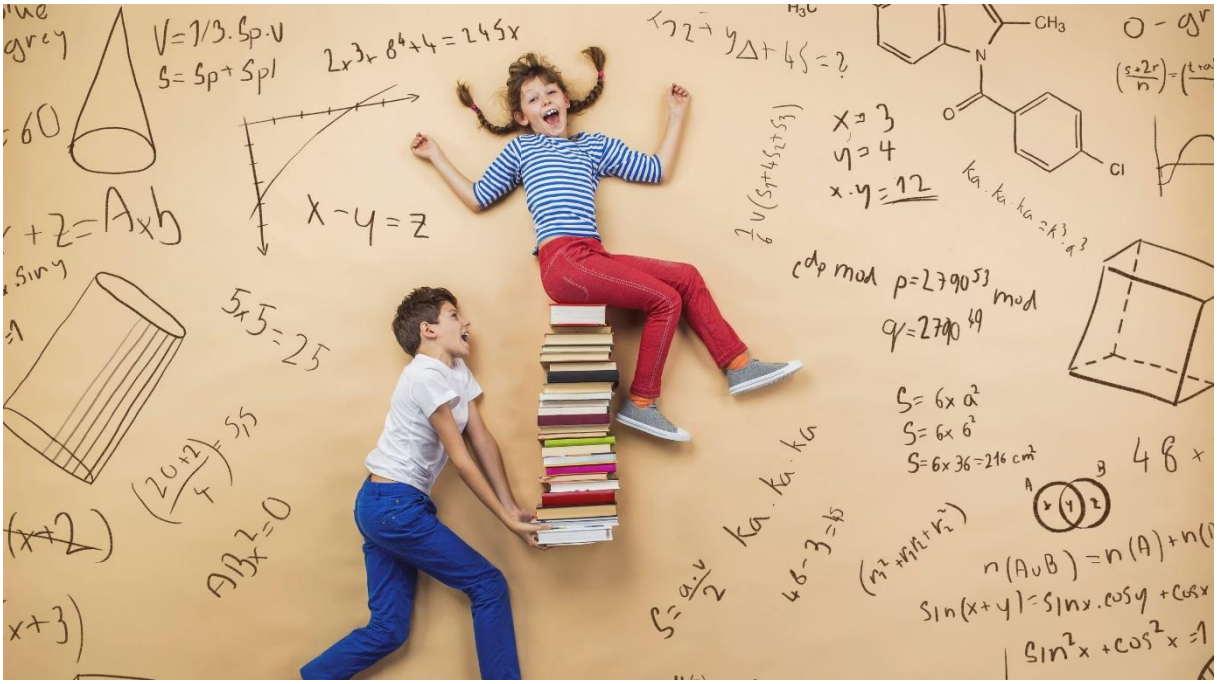
make more conferences like this one !

Mathematics Education -
Relevant, Interesting and Applicable

Newsletter 1, November 2016

What is MERIA?

MERIA is the acronym of the project *Mathematics Education – Relevant, Interesting and Applicable*. The main goal of MERIA is to enhance quality and relevance of mathematics education in secondary schools by using inquiry based teaching and supporting teachers' professional development across Europe. Project MERIA will strive to promote a positive attitude towards mathematics and show that mathematics is engaging, important and useful.



Strategic framework for European cooperation in education and training (ET 2020) addresses, as one of educational targets of EU, reducing the share of low achievement in literacy, mathematics, science and digital literacy across Europe. MERIA project activities address the low achievers in mathematics with the ultimate aim of improving students' achievements and attitudes towards mathematics. Research confirms that most students are unable to acquire basic skills without meeting situations in which these skills are to be used and which are perceived as meaningful by the students. Project MERIA will focus on the development of higher cognitive skills and on the educational paradigm of problem-based learning in upper secondary schools by designing innovative mathematics teaching and learning materials and supporting development of teachers' competencies to establish appropriate "inquiry situations" in classrooms.

For more information visit our website: www.meria-project.eu

Follow us on Facebook: www.facebook.com/meriaproject

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Mathematics Education -
Relevant, Interesting and Applicable

MERIA Team consists of university professors in mathematics and mathematics education, high-school mathematics teachers, representatives of education institutes and experts from mathematics associations. MERIA gathers experts from 11 institutions in four countries:

Faculty of Science (University of Zagreb), Croatia – coordinating institution
Faculty of Electrical Engineering and Computing (University of Zagreb), Croatia
Faculty of Organization and Informatics (University of Zagreb), Croatia
Croatian Mathematical Society, Croatia
XV. Gymnasium, Croatia
University of Copenhagen, Denmark
Vordingborg Gymnasium, Denmark
Matematiklærerforeningen, Denmark
Utrecht University, Netherlands
University of Ljubljana, Slovenia
National Education Institute Slovenia

When does MERIA start?

Project started on 1st September 2016 and it will end on 31st August 2019. Project activities start with a kick-off meeting in Zagreb, on 1st and 2nd December 2016. Partners will join to discuss development of the first results: *Practical guide for inquiry-based mathematics teaching*, *Guidelines for development of innovative inquiry based scenarios and modules, Project impact analysis.*

How can you join?

MERIA Team will design innovative scenarios and modules for inquiry-based teaching and launch a digital repository for teachers across Europe in 2018. Teachers will be invited to use and add materials within the repository and to report on their experience.

Project MERIA intends to support teachers' professional development and offer teacher training. The support for teachers will be offered through workshops on inquiry-based teaching and design of own lecture materials, thus highlighting teachers' "21st century" competences such as creativity and critical thinking. MERIA workshops will be held in partnering countries in spring 2019, followed by an international conference for teachers in Croatia at the end of the project.

Keep updated about the project activities through project's website, Facebook page and newsletters. Newsletters will appear every three months!

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Mathematics Education -
Relevant, Interesting and Applicable

Newsletter 2, February 2017

Why and what is Inquiry Based Mathematics Education?

During the last decades a number of projects in Europe, several funded by EU, have aimed at developing, implementing and assessing inquiry based teaching activities at different levels of the educational system (Artigue & Baptist, 2012; Mass & Artigue, 2013; Ropohl, Rönnebeck, Bernholt, & Köller, 2016). Most of these projects cover mathematics as well as science, where the inquiry approach is more common. Inquiry in science often draws on already sensed experiences, which can be further studied in cyclic processes, which is difficult for many topics in mathematics. Here the cumulative nature of the discipline is a challenge (Artigue & Baptist, 2012). This short text will provide a brief overview of the idea of Inquiry Based Mathematics Education (IBME).

Origins of IBME

Almost a century ago the first ideas were developed that teaching in general should relate to students experiences and should be based on their actions. The educational researcher John Dewey is often associated with the phrase: “learning by doing”. He criticised the structure of curriculum for being the result of years of dealing with and rearrangement of knowledge, which might not fit students’ experiences and even create hindrance for students learning. Rather teaching should revolve around students’ actions (Dewey, 1902). Dewey (1938) emphasised the nature of inquiry and its role in learning and teaching – especially in science. To a large extend he regarded mathematics discourse as the way to order complex data and systematic treatment of outcomes of inquiry processes. The ideas of Dewey have been pursued by several others.

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Within mathematics Felix Klein played a key role in promoting a teaching more akin the activities of research mathematicians and their inquiry processes, which lead to the development of new knowledge. In the beginning of the twentieth century he introduced a reform programme for teacher education promoting practical instructions and the development of spatial intuition (Kilpatrick, 2008). Later George Polya published his book "How to solve it?" (1945), which has been deemed seminal in the problem solving approach to mathematics education. The book describes the problem solving activity, which mathematicians engage in when doing research. Emphasis is put on the role of problems and heuristic competences needed to solve these problems. Heuristic competences cover knowledge and strategies needed to address non-routine problems.

Different approaches to IBME

The solving of none-routine problems are corner stones in several approaches to mathematics education, which was further developed and some initiated in the 1970s: Theory of Didactic Situations (TDS), Realistic Mathematics Education (RME) and Problem Solving as line of research (Freudenthal, 1991; Brousseau, 1997; Schoenfeld, 1992). A shared idea in TDS and RME is, that students should be posed a non-routine problem, which they solve through the development of new knowledge. In TDS this is supposed to happen through adaption to what is called the milieu of the teaching situation (Brousseau, 1997). In RME the development of knowledge happens when students mathematize the phenomena, which the problem address. The theory differs between two notions for this process: vertical and horizontal mathematization (Freudenthal, 1991).

More recent research approaches to mathematics education which can be said to address IBME, have further studied the role of problems and how to make students pose problems. Especially problem posing literature and the Anthropological Theory of Didactics (ATD) address these issues (Singer, Cai & Ellerton, 2013; Chevallard, 2015). But also modelling as part of Mathematical Competence Theory can be said to nurture elements of IBME (Artigue & Blomhøj, 2013; Ulm, 2012).

IBME in MERIA

Research on the implementation of IBME indicate challenges with respect to teachers' certainty in IBME based teaching situations as well as institutional constraints and conditions can favour or hinder IBME initiatives (Hersant & Perrin-Glorian, 2005; Dorier & García, 2013). The MERIA project seeks to overcome these last challenges by designing course activities and a booklet introducing mathematics teachers at secondary level to IBME, RME and TDS, it will provide teachers with predesigned lessons and assist them in further development of RME and TDS based teaching designs.

References: www.meria-project.eu/activities-results/practical-guide-ibmt

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The role of teachers

It is impossible to overestimate the role of teachers in the learning process. The most important factor affecting student learning is the teacher and seemingly more can be done to improve education by improving the effectiveness of teachers than by any other single factor (Sanders et al., 1997). When referring to teachers' qualifications in project MERIA, we accept and emphasize the balance between teacher's subject knowledge and pedagogical knowledge following the academic research and institutional recommendations. To quote the Eurydice report Mathematics Education in Europe Common Challenges and National Policies:

"…concerning highly qualified mathematics teachers, the US National Council of Teachers of Mathematics (NCTM, 2005) states that they must have 'an extensive knowledge of mathematics, including the specialized content knowledge specific to the work of teaching, as well as a knowledge of the mathematics curriculum and how students learn'. In other words, in addition to a 'profound understanding of fundamental mathematics' (Ma 1999, p. 19), teachers must also have what has been coined by Shulman (1986) as 'pedagogical content knowledge', i.e. the practical understanding of how to apply their knowledge and adapt it to their teaching, as well as 'curriculum knowledge', which refers to the knowledge of the content, materials and resources used for teaching, how they are arranged and ways to use them."

MERIA associated schools

Project MERIA is a meeting point for schools, educational agencies, professional associations and higher education institutions. Along with project partners, MERIA collaborates with associated schools in four project countries in obtaining the following specific goals:

1. To promote student-centered, inquiry-based mathematics education. In selected schools, teachers will try new methods and broaden their set of teaching skills. The project started with an in-depth analysis of teachers' needs and barriers.

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2. To create an interactive repository of mathematical showcase materials for secondary schools. Schools with the status of associated partners of MERIA project will be involved in testing and evaluation of the materials designed during the project's lifecycle.
3. To upgrade teaching skills of in-service teachers, so that they have necessary competences for dealing with diversified groups of learners and to teach using learner-centered approaches and methods. Special workshops for teachers will be designed and realized during the project, taking into account information gathered in associated schools.
4. To strengthen cooperation and flow of ideas between educational and research institutions dealing with mathematics education at different levels. Associated schools have a vital role in the project as the main target audience for all activities, as well as valuable partners providing feedback on the design of teaching materials.

List of all associated schools will be available soon on the project's webpage.

Teachers' professional development

Teachers' continuous improvement is an imperative of the profession:

"Following their initial education, mathematics teachers must continue to update their knowledge and skills. Opportunities for teachers to engage in continuing professional development (CPD) can have a substantial impact on their work, their achievement, skills and attitudes as well as on their performance and job satisfaction (Villegas-Reimers, 2003)."

One of the main aims of MERIA project is to provide systematic and overarching support to mathematics teachers in all partnering countries. To accomplish that MERIA team will design new teaching materials and organize a series of workshops to train teachers in using student-centered methods and new materials, as well as how to design their own innovative materials and upgrade their teaching skills. Workshops will be planned and prepared in detail, which will be described in the MERIA Workshop Guide, one of project's six results.

"Teachers of mathematics must not only deliver the curriculum, but they must be able to adapt their teaching methods to the changing needs of students. They must learn to integrate new materials and technologies, and make use of research results relating to student learning and teaching practices in mathematics (Smith, 2004)."

To make MERIA workshops efficient, project team will focus its activities (module design and workshop themes) according to needs of teachers in the partnering countries.

MERIA workshops

There will be two waves of workshops. The first wave will be held in January 2019 and it will cover the topic of inquiry-based mathematics teaching. The second wave will be organized in April 2019 and it will provide teachers with a hand-on experience with the new teaching materials designed according to MERIA principles.

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Mathematics Education -
Relevant, Interesting and Applicable

Newsletter 4, August 2017

How to compare educational systems?

In MERIA we have partners from four partnering countries, The Netherlands, Denmark, Slovenia and Croatia, working together with a common goal – better mathematics education. In order to understand existing systems and to find a potential for improvement in secondary school mathematics education, we have chosen to compare partnering countries in the following segments:

1. Students' performance in mathematics at large scale assessment
2. Types of secondary schools and programs
3. Teaching approaches, methods and materials
4. Teachers' initial education and professional development
5. Assessment of knowledge and skills in mathematics in high schools

Furthermore, analysis of teachers' needs is based on **interviews with teachers in associated schools** in all four countries. Methodology and findings of this analysis will be presented together with the evaluation of the quality and impact of MERIA materials as an output of the project called "MERIA Project Impact Analysis". Preliminary findings show that **teachers want changes in education, but they miss time, resources and freedom to implement IBMT** and manage their classrooms differently.

Students' performance in partnering countries

MERIA especially aims to address underachievement of students in mathematics and thus contributes to the overall aim of the Strategic framework for European cooperation in education and training (ET 2020) to reduce the rate of low achievers from 22% to 15%. According to the **PISA large scale assessment in mathematics literacy**, students from the Netherlands, Denmark and Slovenia perform in Europe's sub-top, whereas Croatian students perform below the OECD average. Of special concern in Croatia is the portion of low achievers of 30.2%, whereas in other countries it is between 13.6% and 16.7%, with OECD average at 23%. If this is considered together with the fact that in Croatia most common teaching approaches can be viewed as traditional with predominant teacher-directed style, one may conclude that it is valuable to investigate the impact of different teaching approaches, e.g. **inquiry based mathematics teaching (IBMT)**, and learn from the differences in educational systems of the partner countries.

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Focus on IBMT

There is much more to be gained for all countries involved. Today's needs of society and professions, especially in highly technological environments, call for different understanding and skills than a few decades ago. Moreover, research studies suggest that retention of knowledge and quality of learning are related to teaching styles (Garner, Garner, 2001; Jukić, Dahl, 2010) in favor of active teaching methods. Research on IBMT confirms that the effects of inquiry-based approaches include benefits for **motivation**, for **better understanding** of mathematics, and for the development of beliefs about the **relevance of mathematics** for life and society (Bruder, Prescott, 2013). Evidence of positive impact of IBMT is especially present for low-achieving students' whose improvement of grades is sizable and persistent (Kogan, Laursen, 2013). Similarly, there is a strong positive impact on girls' learning gains, confidence and desire to persist. At the same time, no harm is done to other students. IBMT also increases students' capacity to **think critically** especially of those students who have not been previously encouraged to think in that way (Hattie, 2009).

Can we implement IBMT?

Curriculum documents in all partnering countries are inclined to promote acquiring of content knowledge through **students' problem solving and exploration**. Having these ideas in mind, MERIA proposes specific approaches for implementing these ideas. However, to get a deeper insight needed for successful implementation of IBMT, the starting point for the project was also to establish the needs analysis of students and teachers. Along with the interviews with teachers in all partnering countries, in Croatia, the analysis was performed on the basis of the questionnaire developed within the Mascil project. It shows that teachers are mostly inclined to implement IBMT in their practices. However, they consider that **neither present curriculum nor the school environment encourages the use of IBMT**. They also feel lack of support in adequate materials and share the concern about the students' attitudes considering that students would maybe feel frustrated by learning through inquiry. Thus, also when Croatia is concerned, teachers support the findings of the Mascil project which suggest that "classroom management, resources and system restriction hinder the implementation of IBMT". Therefore, there is a need for scaled-up professional development for teachers which should address all abovementioned categories of potential restrictions.

The MERIA team has been intensively working on the project's first result: "**MERIA Practical Guide to Inquiry Based Mathematics Teaching**". This booklet is written for mathematics teachers looking for support in understanding what IBMT is and how to implement it. **It will be published in English on 1st September 2017 on the project's webpage!** Translation into Dutch, Slovene, Danish and Croatian will follow.

The booklet and ideas in it will be further disseminated through MERIA workshops. Having in mind that the strength of an educational system lies in its teachers, MERIA aims to start a series of workshops and seminars for the teachers of associated and other schools in order to promote specific methods of implementation of IBMT.

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Mathematics Education -
Relevant, Interesting and Applicable

Newsletter 5, November 2017

What are MERIA scenarios and modules?

One of the results of project MERIA will be a digital repository with scenarios and modules for inquiry-based mathematics teaching. A scenario provides a classroom situation together with a detailed lesson plan that anticipates and describes the actions of both the teacher and the students, while a module also contains additional materials that help to realize the scenario, students' work from previous implementations of the scenarios and various comments on the different phases of the plan.

MERIA scenarios and modules will be designed to provide teachers with clear instructions on how to orchestrate the classroom in order to ensure a potential for inquiry and learning based on a rich situation. These rich situations will be selected with the aim to motivate students and to engage them in activities that have a potential for students to experience that mathematics is relevant, interesting and applicable. The methodology for developing scenarios and modules is supported by two theories – one is the Realistic Mathematics Education and the other is the Theory of Didactical Situations. Both theories are explained in the context of inquiry based mathematics teaching in the booklet of project MERIA called "MERIA Practical Guide to Inquiry Based Mathematics Teaching".

Each scenario contains the description of the target knowledge, broader goals, prerequisite mathematical knowledge, time frames, required materials, the problem and possible ways for students to realize target knowledge. Each lesson phase is described following the practice of the Theory of Didactical Situations.

Test scenario – proportionality and areas

The first scenario that was developed and tested in project MERIA is concerned with the notions of proportionality and areas.

The main problem that students encounter is the following:

Look the picture. If you open it at your smartphone or computer, you can easily drag the picture in order to enlarge it. What happens with the area of the picture covered by the pyramid when we enlarge the picture?

The target knowledge for this scenario is the statement:

Whenever the side lengths of a polygon are enlarged by a certain factor k the area of the polygon is enlarged by the factor k^2.

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The problem and the target knowledge serve as the motivation to engage in different mathematical activities that will enable students to develop broader goals:

Autonomous algebraic and geometric reasoning, student formulation of general statements and proofs based on formulas of perimeters and areas of different shapes, the notion of similar polygons, possibly to include the sine function as well as additivity of area when cutting the polygon in parts. If students are used to work with ICT they could generate hypotheses in a graphical environment and use them as a starting point of a proof.

In this lesson students need to investigate the problem on their own using pen, grid paper, rulers, calculators or ICT. First, they should recall different ways to measure and calculate areas of geometrical shapes. Next, they should gather and organize data and formulate their hypotheses. The main activity of the lesson is for students to formulate their own conclusions, discuss them with peers and to find justification for them.

Of course, different students will work on different levels and with different speeds, so their conclusions will be different as well. The role of the teacher is to encourage each of them, or each group, in their autonomous inquiry and to organize mutual discussion and presentation of the findings. In this way all students build a sense of common ownership and connect mathematics at hand with their own actions and ideas.

Results of questionnaires for teachers and students

After the class with the test scenario, teachers and students have provided feedback through a questionnaire designed for this occasion. The results of 119 students in four schools in Croatia already show that 75,7% of students found the lesson more interesting than their regular lessons, while 47,1% after this lesson have a feeling that mathematics is relevant and connected to everyday life. In their comments students emphasize that the lesson was different than usually because they worked in groups, used computers, and that they had to work on their own. Teachers confirm (in 85,7%) that their students were more active, but they emphasize that the lesson was very challenging for the students and only a minority of students have reached the desired conclusion autonomously.

Further work - meeting in Ljubljana

Encouraged by the positive effects that the scenario had on students' motivation, we believe that further adjustments of the scenarios will make them more suitable for achieving the desired target knowledge, too. The results of the questionnaires from all partnering countries will be analyzed at the next MERIA meeting in December 2017 in Ljubljana. At the same meeting further scenarios will be designed.

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Mathematics Education -
Relevant, Interesting and Applicable

Newsletter 6, February 2018

Meeting in Ljubljana

As we announced in our previous newsletter, the groups from partnering countries have met in Ljubljana, Slovenia. The meeting took place from 11th to 17th December 2017. In this meeting all partners have discussed the effect of the test scenario on the proportionality and areas in different countries and agreed on the focus for the work to be done in the coming months.

The project team analysed the results of the questionnaires filled by the students and the teachers after the implementation of the scenario and possible improvements were suggested. *The most important observation from the scenario is that some teachers tend to teach the students during all phases of the scenario. This ruins the adidactical potential of the situation and strongly distracts students.*

After a discussion about how to asses and evaluate students' work on the scenario, it was concluded to include in the module an immediate task (or more simple tasks) the students are supposed to be able to solve if they have reached the learning goal of the scenario.

First module is published!

MERIA module is an extension of a MERIA scenario. The module consists of the *scenario, explanation of materials*, *variations based on didactic variables, observations from practice, evaluation tools,* and *rationale and RME perspectives on the scenario.*

The module gives suggestions for the teachers on how to use the test scenarios and what to use in a certain phase of the scenario. Possible changes to the scenario are outlined and the importance of what should be left unchanged is emphasized. The student strategies and possible interventions are given with data from the implementation of the scenario.

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If possible, some self-evaluation tools for the students are provided. Some potential strategies are outlined for the teacher so that (s)he might know whether some ideas were left out. The choice of the mathematical knowledge is addressed and also the relevance and applicability of the target knowledge is addressed as a part of a broader goal.

The testing of the first scenario and the provided feedback from the students and the teachers resulted in rich material that forms the first MERIA module. It is published as part of the **MERIA Template for Scenarios and Modules** booklet. *The booklet is now available on project web page. So, take a look and try the scenario yourself!*

New scenarios

A rich open problem for a scenario should encourage the students to engage in asking questions about the given problem and think of a solution. The students should be able to experiment with different ideas they might get. A good scenario would entail different possible strategies that lead to similar (if not the same) solutions. The experimentation process gives the students an understanding of the underlying problem. MERIA scenarios will probably cover piecewise linear functions for modelling optimal production costs, quadratic functions in the physical context of driving a car, introduction to derivatives as the slope of a function, introduction to logarithms as inverses to exponentiation, interplay between algebra and geometry in terms of conflict lines and use of statistics in evaluation job offers.

Further work – new scenarios and modules

In the following months new scenarios will be developed and implemented in classrooms. With the feedback provided from the implementation, the modules for these scenarios will be created by the end of the year.

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Mathematics Education -
Relevant, Interesting and Applicable

Newsletter 7, May 2018

New MERIA scenarios

As we announced in our previous newsletter, new MERIA scenarios have been developed and implemented in classrooms. With the feedback provided from the implementation, the modules for these scenarios will be created by the end of the year. In this newsletter we present the scenario for modelling optimal production costs using linear functions.

Linear functions for modelling optimal production costs

Target knowledge. The construction of piecewise linear functions defined as a solution to a problem where a list of linear conditions is given. In the end the solution could be presented as a piecewise linear function. Graphical calculator or computer can be used to plot functions and solve equations, if students know about that.

Problem. You are a consultant who advises companies on where to run factory buildings for the production of bicycles (or other items), based on the table showing the costs in different areas, what would you in general guide of the companies to choose and why?
The students should write a document explaining to the director of the company how they would advise him to place his factory.

Scenario was written for a lesson of 45 minutes consisting of standard phases of TDS scenario: devolution, action, formulation, validation and institutionalisation. For the scenario, teachers prepared the classrooms for groups of two to four students. At the begging of the lesson, students are given the following table and asked to solve the problem.

Areas of location	Fixed costs for running the factory in €	The costs of producing one bicycle at the factory in €
A	300 000	120
B	450 000	110
C	660 000	60
D	680 000	80

Implementation of the scenario

The Linear functions scenario has been designed for students of age 15-16. The implementation was done in three schools in Croatia, one school in Netherlands and the scenario was also discussed with the students (pre-service teachers) at the Faculty of Science in Zagreb. The analysis of implementation in Slovenian and Danish schools will be done subsequently.

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Students used mobile phones, graphical calculator, Geogebra, Wolfram Alpha or grid paper to obtain graphs of linear functions and calculations.

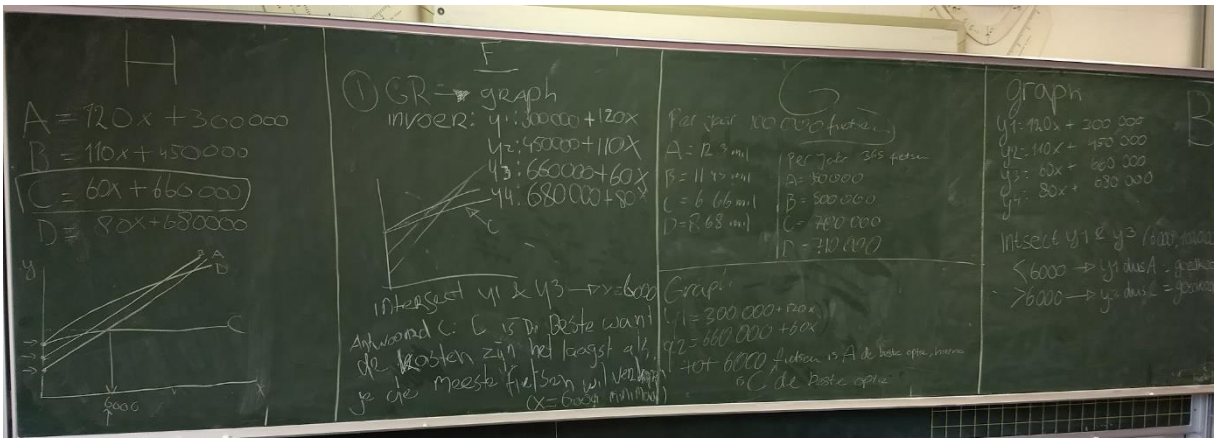
Students had some questions to clarify the problem. Some of them were thinking about the profit, instead of the costs. The students asked: *What are the transportation costs?*
In Netherlands there were no problem with understanding of the problem. The students say: *We have done similar problems at our Economics course, but with two locations. This is very similar.* Some of them realized very fast: *The one with the smallest slope is the cheapest.* According to teachers' suggestion the project team will replace *fixed costs* by *costs of building the factory in that area* in order to clarify the problem.

Further observations

Important observation from the scenario is that the teachers were trying not to teach the students during all phases of the scenario. This is nice improvement to keep the adidactical potential of the situation. During the action phase, students formulated the following approaches:

I. modelling with linear functions and drawing graphs
- I.1. drawing by hand and calculating intersections as solution to linear equations
- I.2. using technology to draw graphs and find intersections

II. Comparing pairs of areas and analyzing the results
- II.1. using linear equations
- II.2. directly from the table, comparing the fixed costs



In the formulation phase students were presenting their strategies on the blackboard in order to learn from each other's strategies in the next phase. According to MERIA questionnaire, after this lecture 73.3% students think that mathematics is related to real life, 87% say that lecture was interesting or much more interesting than usual lecture, and 91.9% students would like to have such lectures each month!

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New MERIA scenarios

In this newsletter we present the scenario in which students are investigating how the braking distance relates to speed just before braking. We find the situation extremely rich as it opens the potential to discuss summation as discrete integration, while at the same time could lead to subtle discussion about average speed in physics.

Scenario Braking distance

Target knowledge: Quadratic relation
Broader goals: Quadratic functions and their characterization by constant second derivative (second differences for quadratic sequences, that is, quadratic functions on integers), or by constant decreasing or increasing first derivative (differences for quadratic sequences).
Interdisciplinary skills: students have to work with variables from physics and make sense of what is going on (bridge the two worlds of notations and procedures).
Inquiry skills: analysing data and looking for patterns in the tables. Justifying their findings (argumentation) during the presentations (the calculations dominate the process and they have to summarize their approach to others).
Problem: In a city area with primary schools, parents complain about the set speed limit, considering it inadequate for the area with school children. A group of reckless drivers say that they do not need to worry because they brake in time. Now you (the students) are asked to investigate how the braking distance relates to speed just before braking.
Consider a car braking in such a way that the speed decreases by 10km/h every 0.4 seconds.
You can use the tables below to organize calculations and make observations, then justify your answer as you best can.

Scenario is designed for two lessons of 45 minutes as introduction into Quadratic function chapter. It contains the standard phases of the TDS scenario: devolution, action, formulation, validation and institutionalization. At the beginning of the lesson, the teacher divides students into groups and poses the problem. In the action phase, students will calculate the braking distance for the concrete initial speeds. In the formulation phase they will generalize their calculations and considerations. During the validation phase the students will present their group's solutions, listen to the presentation of other groups, ask questions and discuss strategies and solutions. The students will discover relationships that are not linear, quadratic

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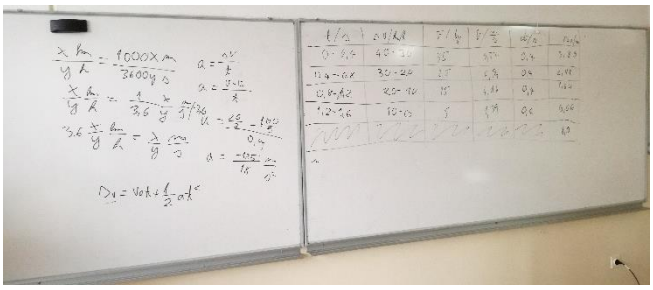
relationships. Finally, in the institutionalization phase, the teacher comments and connects to each other all strategies that appeared in the classroom and introduces the quadratic function.

Implementation of the scenario

Scenario *The braking distance* was tested in two schools in Croatia and one school in Slovenia, and in the Netherlands and Denmark testing will be carried out later.



Students didn´t have problem with understanding the task, and they used different strategies for solving it. Some students calculated the braking distance for concrete speeds, using the proposed tables to organize the data. After that they generalized and got a quadratic relationship. Other students had displayed graphically the obtained data and noticed that the relationship is not linear. Some groups had applied facts and formulas of physics and quickly obtained the required quadratic relationship. The teachers emphasized that the scenario was very interesting, and there was no problem in the implementation, the students were thinking independently and solving the problem. It was noticed that the group that used the facts of physics, solved the task faster than it was predicted. Therefore, instructions were added in the module so that the scenario can be supplemented with new tasks for students who quickly solve the problem.

Further observations

The students pointed out that they had more time for creative approach to the problem, they derived formula by themselves, worked in groups and collaborated more. They were solving the problem task where they used the knowledge of math and physics. They said they like the integrated model of work in math. Describing the lesson in one word they wrote: independent, collaboration, research, physics, interesting, resourceful, creative, great.

The MERIA survey shows that after this lesson, 67% of students think that math is related to real life, 63% of students think that the lesson was interesting or much more interesting than the usual lesson and evaluated it by an average rating of 4.2. Even 93% of students would like to have this lesson at least once a month, and 63% of students at least every two weeks.

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Mathematics Education -
Relevant, Interesting and Applicable

Newsletter 9, December 2018.

Meeting in Zagreb

Another MERIA project meeting and training activity was held in Zagreb from 10 to 16 December 2018. During the training activity, the project team was working on the final versions of project results. We have selected the scenarios to be published and discussed the related teaching modules. Also, project members participated in the implementation of one MERIA scenario in XV. Gymnasium. The main activity of the training week was to discuss and organise the topics and activities for the MERIA workshops that will be held in all partnering countries from January until April 2019. We have also discussed the program for the final conference that will take place in Zagreb in May 2019.

Scenarios and modules

The project team analysed all feedback information on MERIA scenarios received from teachers in associated schools in the period from September until November 2018. Based on this information and the needs in each partnering country, five scenarios were selected to be published as a result called *MERIA scenarios*. These are: *Bicycle Factory*, *Braking Distance, Conflict Lines, Slide* and *Salary.* We have already written about scenarios *Bicycle factory* and *Braking distance* in the previous issues of the MERIA newsletter.

In the *Conflict Lines* scenario, the target knowledge is the partitioning of a plane by perpendicular bisectors of pairs of given points (Voronoi regions). The *Slide* scenario begins with the problem to model the slide with the smooth transition between a curve and a line using tangents. The *Salary* scenario is an introduction to statistical measures of central tendency, so the students will have to determine these measures (arithmetic mean, mode, median) and make decisions based on them.

The MERIA module elaborates on the MERIA scenario and contains additional materials. After the implementation of the scenarios, all information obtained in classrooms were summarized in the modules. Examples of students' strategies and possible interventions will be of use for teachers preparing to implement the scenario.

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Mathematics Education -
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MERIA Workshop guide

Local workshops promoting inquiry based mathematics teaching and supporting teachers wanting to enrich their teaching practice with new methods will be organised from January until April 2019. Guidelines concerning the methodology and materials for the workshops will be published in the *MERIA Workshop guide*. The workshop guide will be flexible enough to facilitate different needs and restrictions in each country. It will support new trainers to perform local workshops with similar activities.

Some of the suggested topics for the workshops are: Inquiry based learning (IBL), The role of the student and the teacher in inquiry based mathematics teaching (IBMT), Theory of didactic situations (TDS) and MERIA scenarios, Realistic mathematics education (RME) and learning trajectories, Experience in implementing MERIA scenarios. For each of them we are preparing workshop materials.

Further work – local workshops

In the following months, we will organise local workshops for teachers in all partnering countries. Teachers will experience inquiry based learning, think about open tasks, about the role of context in RME, learn about phases of a lesson in TDS, try out MERIA scenarios in the role of a student and prepare themselves for the implementation of MERIA scenarios in their own classrooms.

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Mathematics Education -
Relevant, Interesting and Applicable

Newsletter 10, February 2019

We have entered the final phase of the project in which we are disseminating new methods and materials that the project team has been developing for almost two years!

MERIA workshops in Croatia

What a great start of dissemination it was in Zagreb, where more than 40 teachers were trained by the project members in inquiry base mathematics teaching!

The program consisted of more than ten activities throughout two days, on **10th and 11th January 2019**.

Teachers discussed structured and unstructured tasks, mutual expectations of teachers and students, building knowledge on students' results, realistic contexts, theory of didactical situations, phases of a lesson and experienced all five MERIA scenarios. The program ended with a round table with teachers who were involved in the testing of the materials since 2017.

The workshop was held at XV. Gymnasium in Zagreb by Željka Milin Šipuš and Matija Bašić (Faculty of Science) and by Eva Špalj and Sanja Antoliš (XV Gymnasium) with the help of Vesna Županović (FER). After implementation of the scenarios in their own classrooms, teachers will meet again on **6th April 2019.**

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Mathematics Education -
Relevant, Interesting and Applicable

MERIA workshops in Slovenia

We are also excited to share our thoughts on the MERIA Workshop in Slovenia held on **4th and 5th February 2019**! Partners from National Education Institute Slovenia and

University of Ljubljana hand in hand with teachers from the associated schools (Gimnazija Jesenice, Ekonomska šola Novo mesto, Gimnazija Frana Miklošiča Ljutomer) were talking in Rimske Toplice with more than 30 high school teachers. We encourage teachers to upgrade and enrich their teaching practice with IBMT and MERIA materials and show students that mathematics is interesting, relevant and applicable.

On the second day of the workshop teachers were discussing the role of RME through different activities and scenarios, teachers' interventions and the role of structure in a task.

Teachers are planning their activities for implementing IBMT in their classrooms with the help of MERIA scenarios and we will meet again in **April** to discuss the results.

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Mathematics Education -
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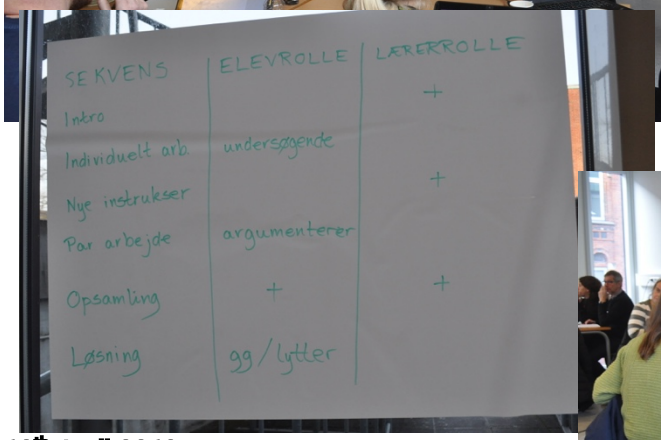
MERIA workshops in Denmark

In Denmark we welcomed 33 high school teachers from all over the country to the first of two MERIA workshop days at Odense Katedralskole on Wednesday the **30th January 2019**. In this first workshop the program included an introduction to inquiry based teaching as interpreted in the MERIA project. There was a series of exercises and a lecture by Britta Eyrich Jessen from the University of Copenhagen, who also participated in the working group of the project. At the end of this first workshop the participants were given a task for the next workshop to test some of the scenarios developed in MERIA and to come up with new ideas for scenarios.



The second day of the workshop will also be held at Odense Katedralskole on Wednesday, **10th April 2019**, when we will continue working on how to develop new scenarios and RME perspectives and finally look at how the Danish curriculum is covered by MERIA.

MERIA workshop in the Netherlands

In the Netherlands there were two MERIA meetings with teachers from associated schools on **4th October and 15th November 2018**. Project members worked with teachers on RME and TDS, and introduced the MERIA scenarios. The next meeting of Dutch teachers participating in project MERIA will be held in Utrecht on **21st March 2019**.

Final MERIA conference will be held on 27th and 28th May 2019 in Zagreb with participants from all four participating countries! Read more about it the next newsletter!

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Mathematics Education -
Relevant, Interesting and Applicable

Newsletter 11, May 2019

MERIA – The final conference in Zagreb, Croatia

We are glad to share great impressions from the MERIA final conference that took place in Zagreb, on 27 and 28 May 2019 as the main dissemination event of project outputs. The conference was hosted by the project leading institution, the Department of Mathematics of Faculty of Science, University of Zagreb. More than 75 mathematics teachers and many members of the project team from all four partner countries participated in the event and contributed to exchange of ideas in stimulating and bright atmosphere. The conference was also attended by researchers in mathematics education from Croatia and representatives of local policy-making institutions.

The plenary part of the Conference was opened by the Croatian Minister of Science and Education, Blaženka Divjak, followed by the Dean of the Faculty of Science, Aleksandra Čižmešija, and the Head of the Department of Mathematics, Igor Pažanin, who all emphasized the importance of stimulating and encouraging approaches in mathematics teaching. Matija Bašić, the project leader, described how it all began three years ago and how the idea behind MERIA evolved and spread.

Project outputs, the MERIA scenarios for inquiry-based mathematics teaching, were introduced to the audience by Gregor Dolinar and Mojca Suban from Slovenia, and the project additional output, a research paper inspired by the MERIA scenario “Slide”, was presented by Rogier Bos from the Netherlands.

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Plenary talks

In her plenary talk, Britta Jessen from Denmark highlighted the benefits of sharing of teacher knowledge and how this idea is embedded in MERIA modules.

Britta and Michiel Doorman from the Netherlands argued also theories that support MERIA scenarios - Theory of Didactic Situations and Realistic Mathematics Education, and questioned the future of inquiry-based mathematics teaching – is it desirable and feasible, can it be made sustainable?

Teachers sharing ideas

Different ways in which mathematics teaching can be designed, while discovering and learning relevant, interesting and applicable mathematics, were shared and discussed among participating teachers in subsequent workshops, pitches and the poster section.

Teachers discussed their experience in implementing MERIA-scenarios, students' reactions to a new instructional format, novelties that this format brought to them or to students, products they considered successful, etc. They also shared their own creative ideas and experience in designing math lessons from everyday practice.

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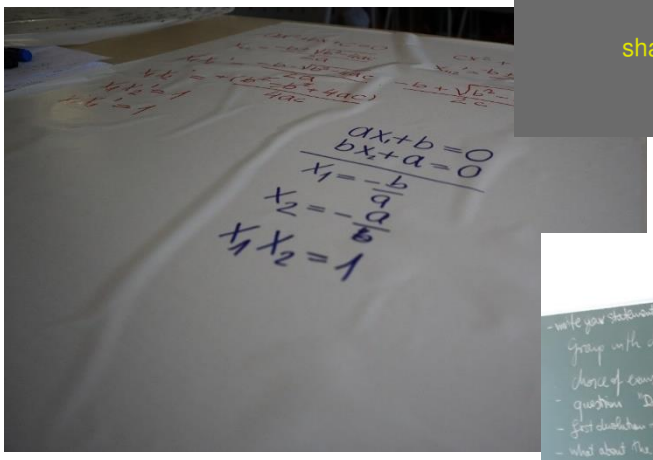
Public session

The last and the most popular (according to evaluation of participants) event at the Conference was the Public session conducted by Eva Špalj from the XV. Gymnasium in Zagreb with participation of students from the same school.



Students worked in groups on a task and elaborated their reasoning and results, after which present mathematics teachers, moderated by Željka Milin Šipuš and Matija Bašić, discussed phases of the lesson, teacher's actions, students' approaches, and other issues observed in the lesson. Enriching experience to all!

We thank everyone for their participation and contributions to the joint work and motivating atmosphere during the Conference!

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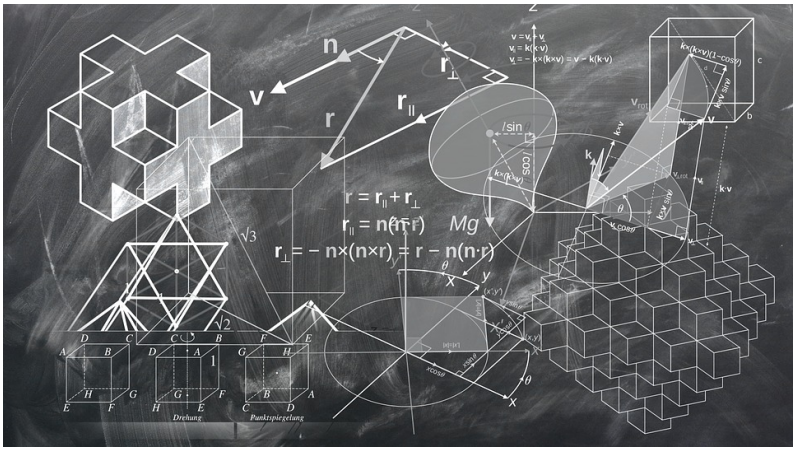
Mathematics Education -
Relevant, Interesting and Applicable

Newsletter 12, August 2019

The results of project MERIA

The focus of project MERIA is to promote inquiry based mathematics teaching (IBMT) across four partnering countries (Croatia, Denmark, The Netherlands and Slovenia), to produce innovative teaching and learning materials and support teachers in the process of implementing new teaching approaches, all with a goal to investigate what can make mathematics teaching more relevant, interesting and applicable to our students.



During the project's lifetime the project team has produced a set of didactical handbooks to facilitate its goals. First is the **MERIA Practical Guide to Inquiry Based Mathematics Teaching** in which the basic tenets of theoretical frameworks are described and illustrated with many examples. One of the main results of the project is a compilation of five **MERIA scenarios and modules** that have been tested and developed in cooperation with teachers in associated schools in all four countries. The scenarios are showcases of didactical situations that provide a very important insight and support for teachers trying to implement inquiry-based mathematics teachers in their own classrooms. Finally, as the change of teaching methods may be quite challenging, we have designed and delivered a cycle of professional development course for around 100 teachers in partnering countries, with all activities described in detail in the **MERIA Workshop Guide.** All of these outputs have been evaluated and improved to support the MERIA team and the final users (mathematics teachers) in achieving the desired goal. Along with online versions available with free access at the project's official webpage, MERIA Practical Guide was published in Croatian financed by the Croatian Mathematical Society. Publishing of MERIA scenarios and modules in Danish has been financed by the Ministry of children and education in Denmark.

Evaluation of MERIA teaching materials and training activities

Quantitively, all targets set out in the application form have been achieved. In total, working with associated schools and through workshops in all four countries, we have introduced MERIA and retrained around 150 teachers. Among these, 31 teachers from 13 associated schools have participated in semi-structural interviews. During the design phase in 2017 and 2018, teachers in associated schools have tested eleven MERIA scenarios in 64 lessons and 1290 students have provided feedback on the relevancy of the lesson and their satisfaction.

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Here are some of the most interesting data from students evaluation.

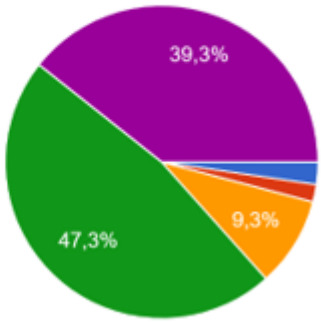
Compare the lesson to a traditional one.

(Sample: Croatia 389 students, Slovenia 306 students, the Netherlands 189 students, Denmark 34 students)

- 1 - Much less interesting
- 2 - Less interesting
- 3 - The same
- 4 - More interesting
- 5 - Much more interesting

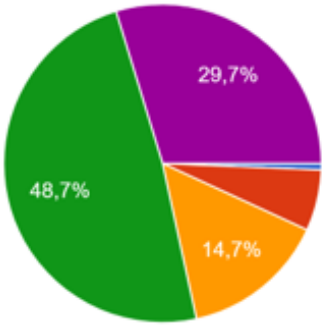
CROATIA

389 odgovora



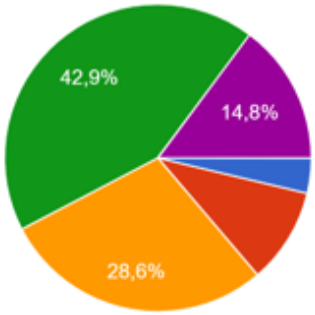
SLOVENIA

306 odgovora



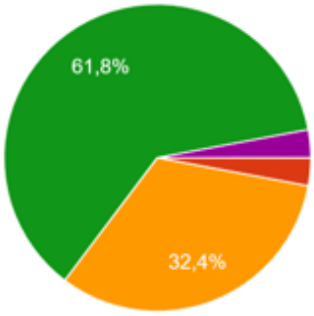
THE NETHERLANDS

189 odgovora



DENMARK

34 odgovora



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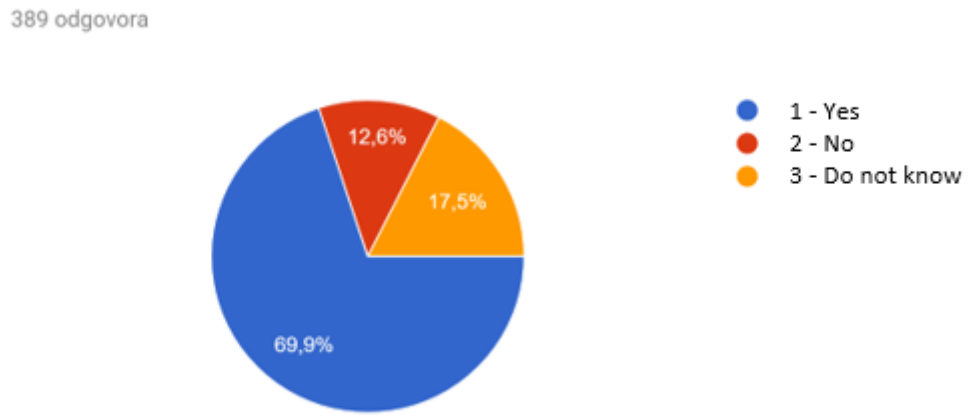
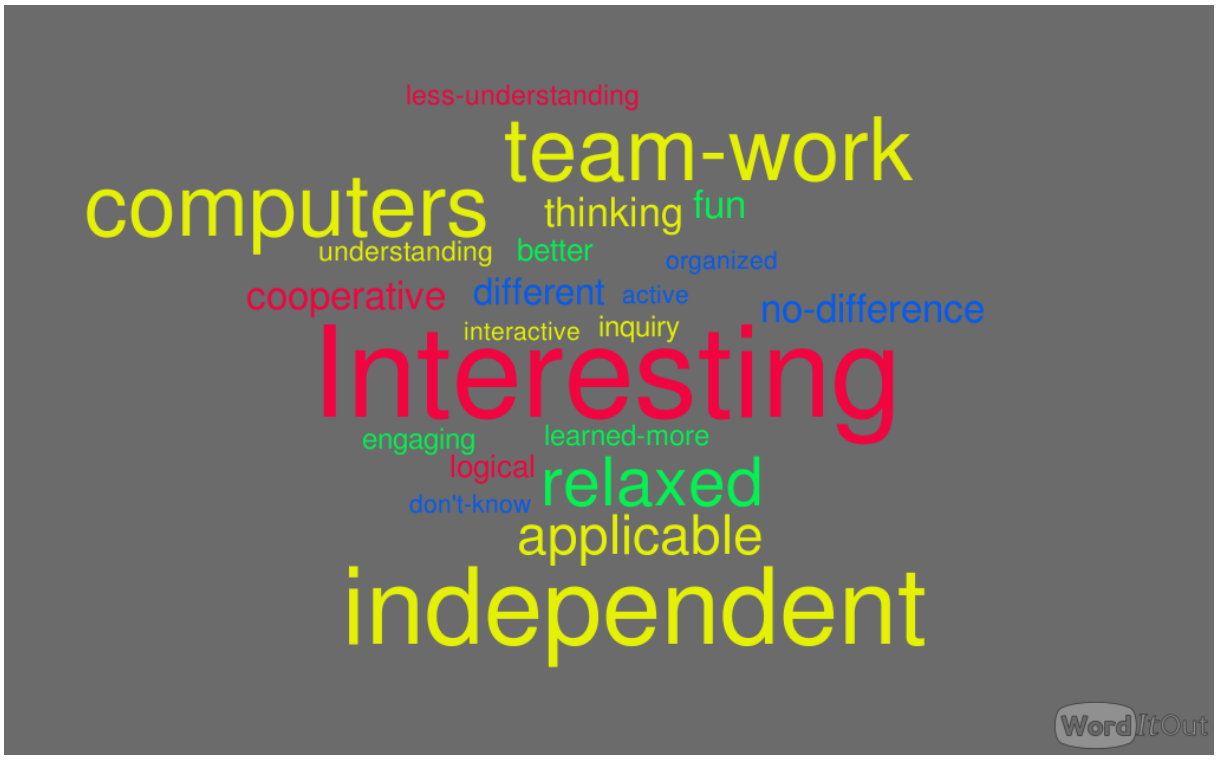
Do you find mathematics more meaningful (related to your life) because of this lesson?
(Croatia, 389 students)



What are the biggest differences in today's lesson and the traditional lessons in your opinion?
(Slovenia, 306 students)



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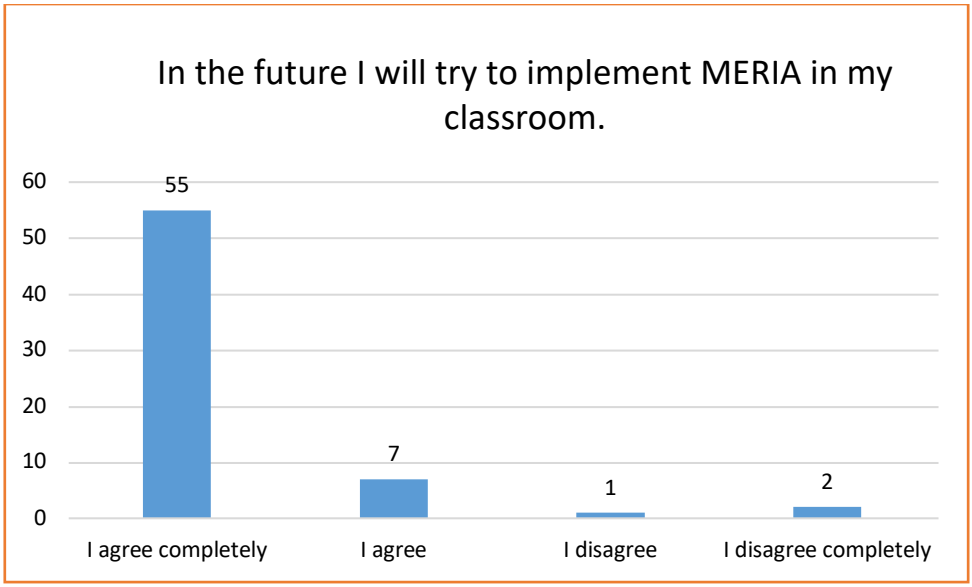
Furthermore, almost all teachers participating in the MERIA professional development cycle were asked to implement at least one scenario and report about the outcome, so we estimate that about 80 more lessons have been performed based on MERIA materials with around 2500 students participating. Here are some of the comments from teachers in Slovenia:

"The content and the material are useful for my everyday teaching practice because they are well prepared, and they contain different ways of students solving the problem. The inquiry method is efficient since it pushes students to start thinking and learn better. The method is very time consuming and we do not always have time. Some examples are too difficult for our students."

"I will plan at least 2-3 MERIA scenarios for the next school year. I will also do shorter inquiry activities because they do not take so much time."

"Cary on what you have started. Spread the idea. Do as many scenarios as you can. The workshop has confirmed my beliefs that we need different instruction and different knowledge. Students can do more than I thought and I will include inquiry activities in my lessons, along with solving open problems. All I need is to make the first step."

And some reactions to MERIA Final Conference organized in May 2019, with around 80 teachers from all four countries.



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I liked the most... the way of thinking, usefullness and importance of the project. Today we have the opportunity to participate in a lot of the lessons and workshops but for the teacher is important to recognize the tree in the forest of this workshops and online lessons.

Suggestions for improvement... Quality before quantity! Thanks!

Reflection on the project's impact

At the end of the project we have repeated the semi-structural interviews with teachers in associated schools. Here are some of the findings, taken from the *MERIA project impact analysis*. Teachers appreciate that they have received detailed materials and an opportunity to discuss the implementation with other colleagues. From the materials they value the structure as it provides them the way how to implement inquiry based teaching, while feedback from other colleagues helps them to gauge their expectations and improve their actions. Teachers' understanding of inquiry based teaching is mostly experiential and they define it as "students working on their own". This understanding has certainly evolved through workshops as they report that their perspective has changed, and they provide examples on it. Concerning their own skills, teachers have improved in their role to "withdraw in a meaningful way" and value the time given to students to work without their interference.

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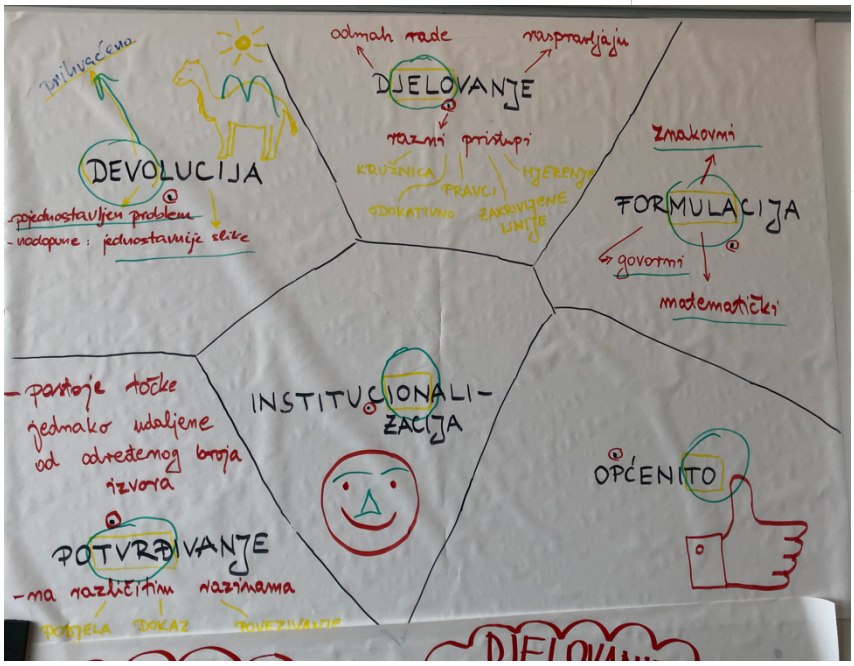

Teachers clearly illustrate their values and support their reasoning with positive examples. They have not adopted the vocabulary of the theoretical frameworks, but it is clear from the interviews that they use the concepts that are central to these frameworks. For example, teachers emphasize students' autonomous actions ("adidactical situations"), they often mention students' expectations and time needed to adapt to a novel way of learning ("didactical contract"), they are aware of the structure of a lesson ("phases in TDS") and of their role to support students ("scaffolding") and to connect students work to the topic of the lesson ("target knowledge" and "institutionalization"), also they value realistic and open problems ("rich contexts in RME" and "didactical potential"). As a form of professional development, they rate MERIA workshops (and the project in general) very positively and emphasize that they highly appreciate being active participants and discussions about their experiences. They point out that there are a few such opportunities as mostly professional development is provided in a form of lecture with no materials.

We have teachers from different types of schools. Some teachers identify their students as "not so good as in other schools" and they find students' lack of prerequisite knowledge as an obstacle to advance to further topics and experimenting with different types of teaching. Some students show negative attitudes towards mathematics and persisting resistance to engage in a discussion or to think about the problems autonomously. Teachers report that students "learn for grade" and describe students' expectations as "show me the procedure, give me another similar task and grade me". Also, all

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teachers reported that during the first implementation of a MERIA scenario students were confused by the change of the teaching method and asked for help what to do. Those teachers that tried more scenarios in the same class report that students adapt successively with time and that there was a high level of satisfaction of both students and the teacher. Teachers claim that their teaching is heavily constrained by the final national external assessment and the amount of the mathematical curriculum it covers, but they clearly point out that logical thinking is also required at these exams and that MERIA scenarios support its development. So, it is useful to incorporate inquiry based teaching since it supports mathematical reasoning required in the final assessment, but this argument is not always clear to students and their parents. This all points that the negotiation of the didactical contract is crucial for successful implementation of inquiry based teaching.

Teachers feel that inquiry situations bring more dynamics to their classrooms and that MERIA scenarios are interesting to their students. They suggest that parents would think that students were only playing and had not learned anything, while they think students work more during inquiry based lessons than while passively copying solutions of tasks. Students get more engaged because of an element of mystery and it was very positive for the teacher that the students realized that behind a guessing game there is no magic, but a mathematical (scientific, logical) explanation. In particular, students like new ways to present their results (e.g. in the form of a letter to a mayor, or when multiple students write on the board at the same time) and working in a relevant context (discuss payrolls, provide advice, construct a slide). Overall, all teachers report that students were engaged and that they were positively surprised by the students with more modest achievements. In general, teachers feel that they have enough time and freedom to incorporate inquiry based lessons during the year and add that this statement is related to the fact they have tried the scenarios ("now that I know what it is about") and received positive feedback ("my students managed and were satisfied"). So, we conclude that it is personal experience that leads teachers' decisions. It is also important to note that teachers express positive attitudes towards the materials because they trust the authors (the authority of the project team). Hence, we further hypothesize that the use of materials and the success of the professional development also depends highly on the persons providing it.

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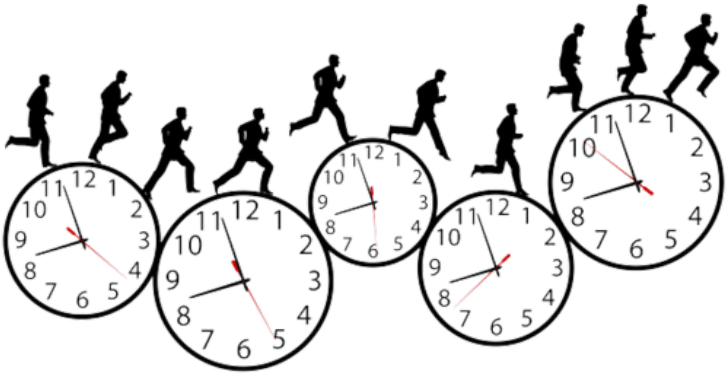
Mathematics Education -
Relevant, Interesting and Applicable

Follow up – Teachers' Inquiry in Mathematics Education (TIME)

The story does not stop here! The same project team will continue to provide further professional development cycles and we hope that more scenarios will be produced in the near future. As we learned much from MERIA and all of the teachers that were involved, a need for a follow-up project emerged.



The main idea of the project Teachers' inquiry on mathematics education (TIME) is to explore how a community of math teachers working together in one school can improve their practice through joint exploration, planning and interaction between themselves and with university professors. We will explore how such a community can reach a high level of independence and sustainability in creating innovative teaching and learning materials and upgrade their professional knowledge.

More concretely, the idea is to adapt the well-established format of Lesson study (LS) to conditions in the participating countries. LS is a Japanese model of shared inquiry, performed by small groups of teachers with a specific goal to improve their teaching, and thereby students' learning. In this project we propose to form LS teams of teachers in all partner countries. The teams will work on implementation of inquiry based mathematics teaching (IBMT). While there is an abundance of research showing effectiveness of LS at the primary and lower secondary level, especially in Japan and other East Asian countries, the model is new in Europe and especially at the upper secondary level.

The project TIME will draw on results and networks established by MERIA. MERIA results show that teachers are eager to learn about IBMT, and that innovative scenarios based on Theory of Didactic Situations and Realistic Mathematics Education produce inspiring lessons which motivate students and allow them to engage in higher level mathematical activity. MERIA also showed that design of such materials is an iterative process that requires expertise and time. Dissemination activities showed that teachers need such innovative materials and moreover that they often do not believe they can produce them on their own.

The first part of the project (TIMELESS) is dedicated to establishing LS among all partners. The second part (TIME^2) will shift the focus to teachers taking the role of designers, as it is the usual practice of independent Lesson study. In particular, teachers will learn how to use didactical phenomenology to develop new scenarios with rich and motivating mathematical contexts. During dissemination activities at the end of the project, experiences and materials will be shared to a wider public of teachers. Stay tuned!

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