Mathematics Education -
Relevant, Interesting and Applicable

MERIA WORKSHOP GUIDE

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Project MERIA, August 2019.
www.MERIA-project.eu

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Introduction

This workshop guide is based on workshops we organized within the Erasmus+ project MERIA to promote inquiry-based mathematics teaching. The guide supports the implementation of professional development programs across the participating and other countries. The way of working is based on cycles of analysis-implementation-reflection so that teachers' knowledge and professional competences related to inquiry-based mathematics education will be extended. The workshops both have an informative character and provide participants with hands-on experiences.

The workshop activities:

- promote inquiry-based mathematics teaching in the participating teachers' classrooms,
- introduce thought-out practices that illustrate inquiry-based mathematics teaching,
- enable participants to reflect on (MERIA) classroom materials for IBMT, and to develop their own materials, adapted to their daily practice, and
- support participants reflecting on their experiences with IBMT-materials and teaching methods.

This workshop guide provides suggestions for organizing and structuring the workshops, guidelines for the workshop instructors and materials to present and use during the workshops. The guide is organized by activities on inquiry-related topics:

1. Inquiry-based mathematics teaching
2. Role of students and teachers in IBMT
3. Structure for IBMT lessons
4. Inquiry as a part of learning trajectories
5. Experiencing MERIA scenarios

Furthermore, the activities in this guide are supposed to offer a flexible set of resources that can be adapted to each country's needs. The guide can also be used for educating new IBMT-workshop instructors.

The workshop activities

The activities in the workshop guide refer to several resources, among which the most important ones are:

- The MERIA practical guide[1],
- MERIA Scenarios & modules[2],
- PowerPoint presentations and other resources per country

A summary of the available activities is listed in the following table:

Topics	**Activities**	**Aims**	**Time**	**Part**
1. Inquiry based mathematics teaching (IBMT)	Introducing IBMT	Let participants experience inquiry, reflect on inquiry processes and on how this was organized	30 min	First
	Tasks that support IBMT	Become aware of what students learn from structured and from more open tasks	60 min 30 min	Any
	Results of inquiry	Experience rich results of students' inquiries and learn to see valuable elements for further teaching	30 min	First
2. Role of students and teachers in IBMT	Dealing with open questions	Become aware of students' interpretations of an open question	30 min	First
	Interventions and how to withdraw productively	Experience how to withdraw in a-didactical phases, how to intervene and scaffold students learning	45 min	Any
3. Structure for IBMT lessons	Introducing the Theory of Didactical Situations (TDS)	Experience and introduce phases of TDS.	60 min	First

[1] https://meria-project.eu/activities-results/practical-guide-ibmt
[2] https://meria-project.eu/activities-results/meria-teaching-scenarios & https://meria-project.eu/activities-results/meria-teaching-and-learning-modules

	Introduction to MERIA scenarios	Experience the potential of MERIA scenarios and become inspired to use them	60 min	First
4. Inquiry as part of learning trajectories	Emergent models in Realistic Mathematics Education	Learn how student work can be the starting point for developing formal mathematics	60 min	Any
	The role of contexts in RME	Become aware of the various roles of contexts, in particular on potential for learning.	30 min	Any
	Mathematics education for RIA mathematics	Become aware of broader goals of mathematics education, like interesting, relevant and applicable, and to what extent these can be fostered by MERIA scenarios	30 min	Any
5. Experiencing MERIA scenarios	Introducing MERIA modules	Introduce MERIA modules	60 min	First
	Preparing for workshop homework	Prepare participants for using scenarios as homework	30 min	First
	Reflection on the implementation of MERIA scenarios	Participants share the experience of implementing a MERIA scenario and compare their differences	60 – 90 min	Second
	Including previous experiences	Participants learn from experienced participants how to implement scenarios	30 – 45 min	First
	Sustainable innovation	Exploring ways of sustaining this way of working in your school	15 – 30 min	Second

Each of the activities is described by first stating the aim of the activity, the prerequisites and time needed and the main issue that is addressed. This introduction is followed by an outline of the activity and a description of the instructor's actions. Each activity ends with some suggestions for further study and possible follow up activities.

The list of themes and the order of the activities suggest an order for the workshops. However, instructors are free to adapt the activities and the suggested order. We are fully aware that in most countries and situations not enough time is available to do all activities. Instructors are invited to choose the activities that fit their needs within the conditions of the workshop(s) they organize. In addition, activities can be adapted or used as source of inspiration. Please refer to the original documents as long as your practices fit the MERIA spirit of aiming at organizing classroom situations in which students inquire and that promote mathematics being relevant, interesting and applicable

The structure of a workshop

The activities the guide provides should be organized into a workshop according to the local needs and time available. The fourth column of the above table provides an estimated duration. Workshops could have two or three parts. The advantage of three parts is that participants could have two rounds of trying out IBMT-lessons, the second time bringing to practice what they learn from sharing and reflecting on part 2. The fifth column indicates in which part an activity should be held, if there are two parts available. Some activities could be repeated, in case of a three-part workshop.

In the table below we present sample schedules for the first and second part of the workshop, as trialed in Slovenia and Croatia:

Schedule first part of workshop	
Slovenia	Croatia
1. Introducing IBMT 2. Introducing TDS 3. Introduction to MERIA scenarios 4. The role of context in RME 5. Results of inquiry 6. *Emergent models in RME* 7. Tasks that support IBMT 8. *Interventions and how to withdraw productively* 9. Including previous experience 10. Preparing for workshop homework	1. Introducing IBMT 2. Tasks that support IBMT 3. Result of inquiry 4. *Dealing with open questions* 5. The role of contexts in RME 6. Introducing TDS 7. Introduction to MERIA scenarios 8. Preparing for workshop homework 9. Including previous experiences.

Schedule second part of workshop	
Slovenia	Croatia
1. Including previous experiences 2. Reflection on the implementation of MERIA scenarios 3. Dealing with open questions 4. Sustainable innovation and a pitch of new ideas 5. Evaluation and feedback on the workshop	1. RME - Mathematics – RIA 2. Emergent models in RME 3. Reflecting on the implementation of MERIA scenarios 4. Introducing MERIA modules 5. Sustainable innovation

In italics we indicated what activities were unique choices for that countries' first part of the workshop. The minimum to do in the first part is: Introducing IBMT, Introducing TDS, Introduction to MERIA scenarios and Preparing for workshop homework. This takes an estimated time of three hours. The main difference between the two sample schedules is that Slovenia takes the shortest route to the MERIA-scenarios, so that they can function as a reference for the rest of the workshop, whereas in Croatia they chose to take a slower approach, first emphasizing general aspects of IBMT. Both have shown to be good options.

1. Inquiry based mathematics teaching (IBMT)

Workshop activity: Introducing IBMT

Aim	Experiencing an IBMT activity, introducing phases of inquiry, reflecting on existing beliefs and practices regarding IBMT in mathematics
Prerequisite	None
Time	30 minutes
Required material	Ppt, post-its and black/white board
Main issue: Inquiry based learning originates from science education and connects to doing practical assignments. In this activity the participants become aware that inquiry processes are present in the practice of mathematics and can be addressed in educational practices. The participants familiarize themselves with the phases of inquiry, how these can be addressed in mathematics education, and to what extent this relates to their daily practice.	
Task description	The instructor presents a question to the participants that seems simple, is easy to start with, but asks for some reasoning before getting some results: *In what numbers of squares can you divide a square?* The participants start working on the problem (can be in small groups) and quickly will feel the need to be clearer about the question and what is allowed. After a few minutes the instructor organizes a plenary to ask how they interpreted the task and what actions are possible. A division in 4, 9 and 16 are easy ways, try also to let the participants realize that 7 is possible. Use that to sharpen the rules. E.g.: squares do not need to be of equal size, and you don't count squares that embrace a division of other squares. The instructor lets the participants continue to work on the problem for circa 10 minutes. In the end, hopefully some will find that when n is possible, then also n+3 is possible (replacing a square by its division in 4). And since 1, 4, 6, 7 and 8 are possible, a square can be divided in every number of squares, except for 2, 3 and 5 (which still needs a proof…). After sharing the result, the instructor asks the participants to discuss the process of inquiry they have been involved in and write down their comments on post-its in the following categories: "Processes in problem solving", "Advantages of IBMT", "Disadvantages of IBMT" and "Questions". The instructor will look at these

	comments during the day and try to address the arisen issues in the following activities.
Instructor's actions	The instructor initially poses the problem without any information, but guides the inquiry by the participants. In reflecting on the solving process, the instructor orders and summarizes the post-its on the wall. The instructor writes on the board the processes that participants recognized (e.g. trying to understand the problem, drawing possibilities in an unordered way, recognizing patterns and becoming more systematic, finding general rules). The instructor connects these processes to phases of inquiry: questioning, exploring, hypothesising, experimenting systematically, generalizing, communicating results, posing new questions (how to prove that 5 is impossible?). The instructor pays attention to connect the task to the curriculum by pointing on the relation with arithmetic series (e.g $x_{n+1} = x_n + 3$ as a description of dividing one of the present squares in four new ones). The instructor introduces the IBMT cycle and asks participants how and when the involved phases of inquiry get attention in their daily practice. All discuss opportunities and limitations of IBMT in mathematics education. Probably one of the issues is the kind of tasks provided by textbooks.
Further study	Practical guide on IBMT. Next suggested activity: Tasks that support IBMT

Workshop activity: Tasks that support IBMT

<table>
<tr><td>Aim</td><td>The participants become aware that most textbook tasks are very structured and provide precisely the strategies or techniques to use. They learn to recognize IBMT task-characteristics and become aware of a changing role of the teacher when implementing such tasks. They experience important phases in lesson plans for implementing IBMT-tasks.</td></tr>
<tr><td>Prerequisite</td><td>Introducing IBMT</td></tr>
<tr><td>Time</td><td>60 minutes (+30 minutes reflection on try-out at next session)</td></tr>
<tr><td>Required material</td><td>Ppt and worksheets</td></tr>
<tr><td colspan="2">Main issue: Students need opportunities to experience and learn processes of inquiry in mathematics. Most textbook tasks structure the thinking and problem solving process for the students. In IBMT, and in order to create opportunities for inquiry by students we need more open tasks that do not have standard solution procedures. This can be achieved by unstructuring textbook tasks. When implementing such open or unstructured tasks the role of the teacher changes (otherwise students might get lost quickly or might go into unproductive directions). Motto of the activity: take the structure out of the task and put structure into the lesson plan. The emerging phases of lesson plans for implementing these more open tasks will anticipate on the didactical and a-didactical situations in TDS and in MERIA scenarios.</td></tr>
<tr><td>Task description</td><td>First ask the participants to work through one of the structured problems carefully (A or C) and list all the decisions that are being made for the students. Next, compare the less structured versions of the task (B and D) with the structured version and discuss the decisions left to the students, and the pedagogical issues that will arise when implementing less structured tasks.

In the discussion with all participants distinguish between the learning of the students and the teaching issues for the teachers. Issues that participants might raise are:
- It is more difficult to plan a lesson with these problems.
- Students may not even know how to get started.
- Students will not necessarily use what they are taught.
- If we offer help too quickly, students will simply do what we say and not think for themselves.
- Students will generate a great variety of approaches and solutions.
- Students may need reassurance that it is OK to try a different approach or reach a different conclusion.</td></tr>
</table>

	Let participants try to jointly come up with ways to deal with obstacles they see. Next, try to 'resolve' (most) issues being raised. Finally, let the participants choose one of the two unstructured tasks and give them time to describe how to: - Organise the classroom and the resources needed. - Introduce the problem situation. - Explain to students how you want them to work. - Challenge/assist students that find the problem too straightforward/difficult. - Help them share and learn from alternative problem-solving strategies. - Conclude the lesson. Discuss the suggested lesson plans and anticipate on TDS by identifying successive lesson phases. Let the participants try-out their plans before the next session (and report in advance).
Instructor's actions	See the task description.
Further study	See the Practical guide on IBMT, especially the variations for the Drug level task that specifically create opportunities for students to experience the inquiry process *questioning* or the process of *planning* in mathematics. This activity is an adaptation of the PRIMAS PD module on Tackling Unstructured Problems. See: https://primas-project.eu/modules/modules-english/

Worksheet: Tasks that support IBMT

Sample tasks
Task A A patient is ill. A doctor prescribes a medicine for this patient and advises to take a daily dose of 1500 mg. After taking the dose an average of 25% of the drug leaves the body by secretion during a day. The rest of the drug stays in the blood of the patient. How much mg of the drug is in the blood of the patient after one day? Finish the table.

Day	Mg of drug in blood
0	0
1	1125
2	
3	

Explain why you can calculate the amount of drug for the next day with the formula: new_amount = (old_amount + 1500) * 0,75
After how many days has the patient more than 4 g medicine in the blood? And after how many days 5 g?
What is the maximum of amount of the drug that can be reached?

Task B

A patient is ill. A doctor prescribes a medicine for this patient and advises to take a daily dose of 1500 mg. After taking the dose an average of 25% of the drug leaves the body by secretion during a day. The rest of the drug stays in the blood of the patient.

Investigation

Use calculations to investigate how the amount of the drug (in mg) changes when someone starts taking the drug in a daily dose of 1500 mg with for instance three times 500 mg.

Are the consequences of skipping a day and/or of taking a double dose really so dramatic?

Can each amount of drug in the blood be reached? Explain your answer.

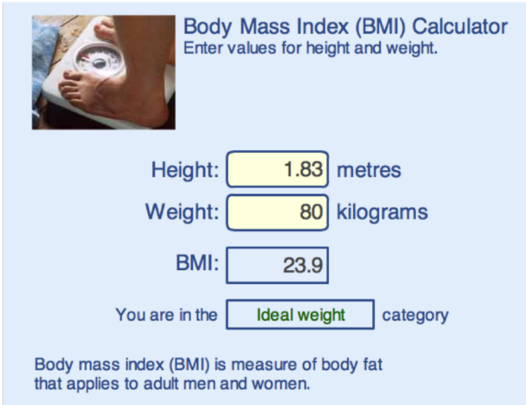
Product

Design a flyer for patients with answers to the above questions. Include graphs and/or tables to illustrate the progress of the drug level over several days.

Task C

Calculating Body Mass Index

This calculator is used to help adults find out if they are overweight.



1. Fix the height at 2 metres - a very tall person!
Complete the table below and draw a graph to show your results.

Weight (kg)	60	70	80	90	100	110	120	130	140
BMI									

(a) What is the largest BMI for which someone is underweight?
(b) What is the smallest BMI for which someone is overweight?
(c) When you double the weight, what happens to the BMI?
(d) Can you find a rule for calculating BMI from the weight?

2. Fix the weight at 80 kilograms and try varying the height.

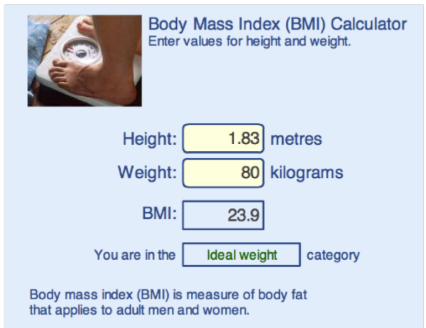
(a) When you double the height, what happens to the BMI?
(b) Can you find a rule for calculating BMI from the height?
(c) Draw a graph to show the relationship between the height and the BMI.

Task D

Calculating Body Mass Index

This calculator shown is used on websites to help an adult decide if he or she is overweight. What values of the BMI indicate whether an adult is underweight, overweight, obese, or very obese?

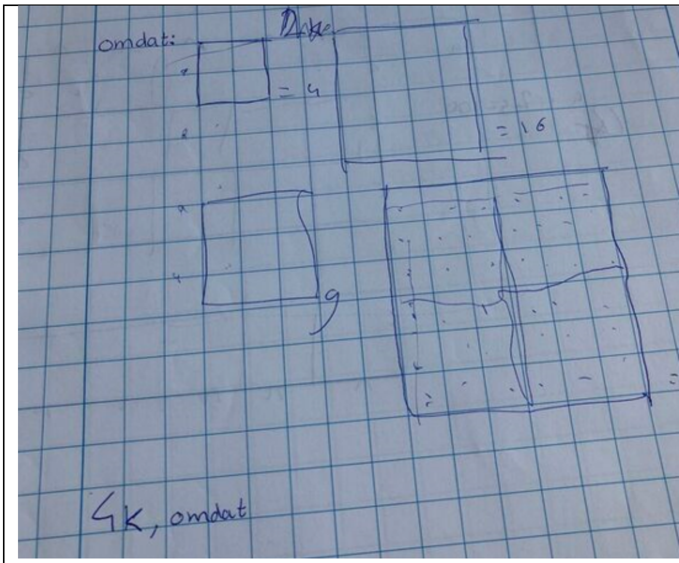
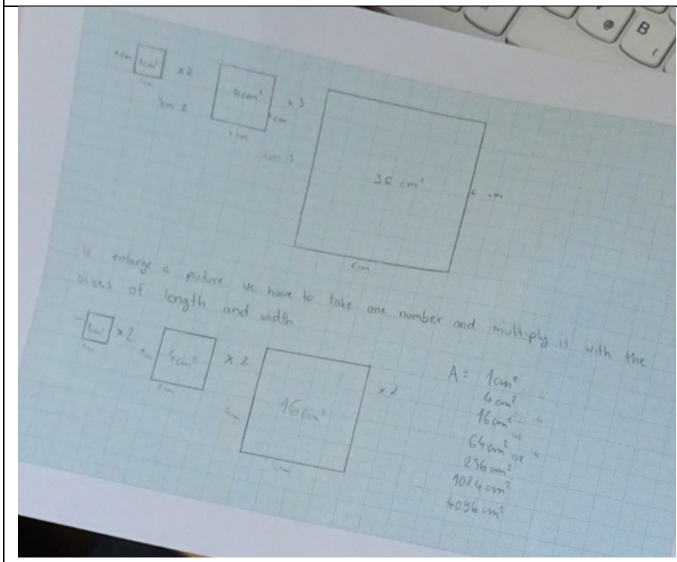
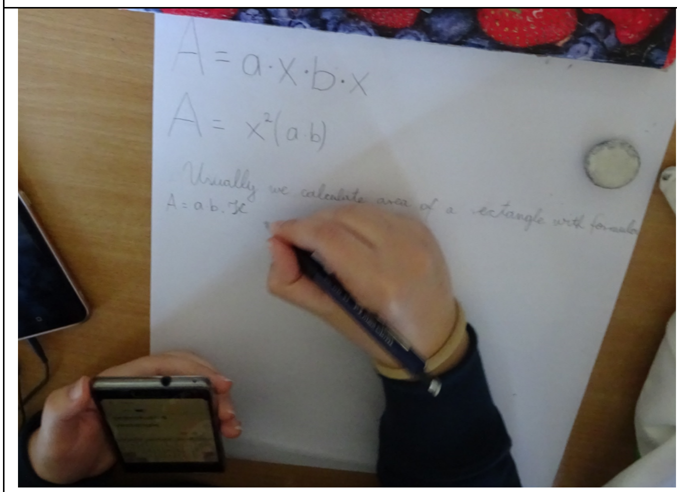
Investigate how the calculator works out the BMI from the height and weight.

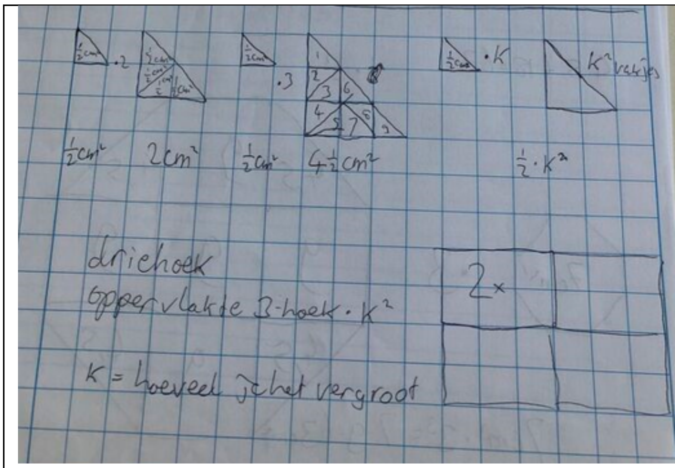


Note for pupils: If you put your own details into this calculator, *don't take the results too seriously!* It is designed for adults *who have stopped growing* and will give misleading results for children or teenagers!

Workshop activity: Results of inquiry

Aim	Getting a first idea why it can be interesting and useful to work with student's own inquiries. Participants learn to appreciate incomplete solutions and unsuccessful strategies and find valuable knowledge and skills in them.
Prerequisite	None
Time	30 minutes
Required material	Worksheets with the sample students' answers
Main issue: Participants may wonder how a students' inquiry can contribute to their learning trajectories. By discussing an inquiry task and investigating student work on the task, this activity aims to introduce an understanding of the role inquiry can have in learning trajectories.	
Task description	An inquiry task is presented. Participants are first asked what they expect their students would do with the task. Then some existing students' work is studied and compared to the participants' expectations.
Instructor's and participants actions	The instructor presents the task from the MERIA Area-scenario:   **Problem:** Look at these two pictures. If you open them on your smartphone or computer, you can easily drag the pictures in order to enlarge them. But what happens with the areas of the pyramid and of the black building when we enlarge the picture? The instructor asks the participant to write down some approaches they expect their students to take (5 minutes). Participants are asked to present some of these approaches (5 minutes). Participants study the sample student approaches (10 minutes). Participants should think what to do when they see this variety in their class: what next? The instructor and participants discuss the findings. (10 minutes)
Further study	Area module
List of additional materials	Area module Mascil Teacher guide: Drug concentration (http://www.fisme.science.uu.nl/toepassingen/22038)

	This one might be easily dismissed as wrong, but actually the students only looked at doubling the side. So, the "4" in "4k" does probably refer to a hypothesis: if I double the side, the area quadruples. Unfortunately, another mistake is made: k is taken to stand for the area of the original square, whereas in the assignment it is used as the enlargement factor.
	These students have some pictures, a description of what happens and a table. It is incomplete, since there is no solution of the general problem. Anyway, these partial results may be worth presenting as a starting point for further discussion, or as data to validate other students' findings (if they do have a formula).
	This is an interesting finding for rectangles. The teacher could ask (all) students whether they agree with this reasoning and if it applies to other polygons as well.

	These students have a hypothesis, but it is not clear whether they have any general argument to support or prove it. The teacher should ask for their reasoning, see if other students might be able to give arguments, and ask whether this reasoning can be applied to other polygons (like the rectangle in the picture).

2. Role of students and teachers in IBMT

Workshop activity: Dealing with open questions

<table>
<tr><td>Aim</td><td>To become aware of students’ issues in interpreting an open question.</td></tr>
<tr><td>Prerequisite</td><td>Introducing IBMT</td></tr>
<tr><td>Time</td><td>30 minutes</td></tr>
<tr><td>Required material</td><td>Worksheet with tables/questions</td></tr>
<tr><td colspan="2">Main issue: Teachers are not always aware of the issues students have in understanding what is expected, what they have to try to do, when the problem is (completely) solved. It is important to discuss these issues with students.</td></tr>
<tr><td>Task description</td><td>The instructor poses a “difficult” problem, with an ill-formulated question, e.g.

Dimension of Koch Snowflake is approximately 1.26. What do you think?

Participants in pairs have 2 minutes to think and answer the question. Instructor will then ask:
What was your task? Was it clear? How did you feel?
Think-pair-share (discussion) on those questions. The discussion should lead to the point where it becomes clear that inquiry problems can be formulated in an open way, but what is expected from the students should still be clear.

The main task focusses on the formulation of inquiry tasks. Participants are asked to discuss the (type of) questions on the worksheet. The question to discuss is

Will the teacher’s expectations be clear to the students?

If there is a gap between teachers’ expectation and students’ possible answers, participants should discuss how to bridge and mend it.

Issues that participants might raise are that in spite of the teachers’ expectations…
…students will be satisfied in finding one solution without trying to find other.
…students do not understand the importance of formulating their findings.
…students are not familiar with creating drawings for trying to better understand the problem and/or potential solution strategies.</td></tr>
</table>

	• ...students don't like the inquiry situation, for example because they feel vulnerable, exposed or uncertain; they do not like to share uncertain ideas to others. • ...students do not feel the need to generalize. • ...students do not feel the need for a proof. • ...students do not feel they have to reflect. Conclusion: The task instruction should not only address the mathematical task, but also suggest on the desired activities of students.
Instructor's actions	Posing the tasks and questions, leading the discussion
Further study	Practical guide

Worksheet: Dealing with open questions

Problems posed to students	**What is the solution?**	**What do you think is a satisfactory solution for your students?**
Find all points with the same property!		
Investigate the relation between a and b		
Can you find the pattern …		
In what number of squares can you divide a square		
What happens with the area of the pyramid when we enlarge the picture?		
Describe the story using a mathematical model and explain it by mathematical means.		
Use mathematics to design a shape with a certain property!		

Workshop activity: Interventions and how to withdraw productively

<table>
<tr><td>Aim</td><td>Experiencing the difference between withdrawing in a-didactical phases and intervening and scaffolding students learning in didactical phases.</td></tr>
<tr><td>Prerequisite</td><td>Tasks that support IBMT</td></tr>
<tr><td>Time</td><td>45 minutes</td></tr>
<tr><td>Required material</td><td>Videos, student's work</td></tr>
<tr><td colspan="2">Main issue: The role of the teacher in IBMT is sometimes didactical and sometimes a-didactical, i.e. sometimes the teacher refrains from explaining and providing mathematical help. It is a challenge for teachers to navigate between being didactical and a-didactical. This activity addresses this issue.</td></tr>
<tr><td>Task description</td><td>Participants watch a few short videos showing students in action and teachers intervening in different ways. Videos are stopped after we see students' action, but before the intervention. Participants have to think about their interventions.

Concrete instructions:
- Watch the video and comment on what you see in pairs.
- Try to find out what is the main question that students encountered and struggled with.
- Find your own intervention (reaction) to their question/problem. A discussion in plenum follows with the questions by the instructor.</td></tr>
<tr><td>Instructor's actions</td><td>Instructor explains the task. He shows the video and stops the video in the moment after the students' action is observed, but before the intervention of the teacher.
Instructor invites participants to identify the problem/question and prepare their intervention according to the identified problem.

The discussion follows regarding the following questions:
What was the main question/problem of the students?
What would you do? Why? In which way would your intervention help the students to move on?
What do you think would be the reaction of the students?
What else could you do?
Would your intervention be different for different groups of students?

Participants in pairs report on their interventions and comment on other interventions.</td></tr>
</table>

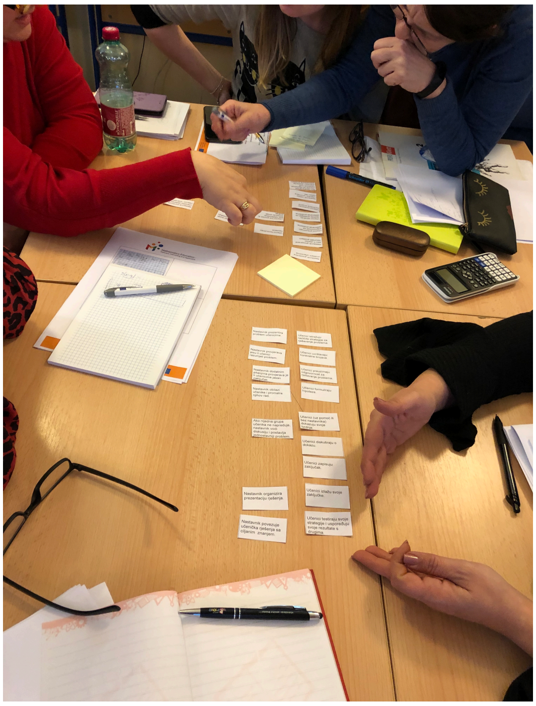
	Instructor starts the video, participants observe the reaction of the teacher and the students and comment on it. Discussion follows regarding the following questions: In what way was the teachers' reaction useful for students? In what way it helped them to move forward? What was the reaction of the students? Would you change your intervention after seeing the second part of the video? Why? Instructor shares the worksheet with the participants for further reflection.
Further study	
List of additional materials	Ppt, students work, classroom video from participants. If not available you can use a video of an IBL lesson from the Primas-project: https://www.primas.mathshell.org/pd/modules/1_Student_led_inquiry/html/videos_2.htm?lang=EN (e.g. stop it halfway when the one of a pair of boys says: "So it never stops. It just keeps tessellating"). Worksheet "Teacher's interventions"

Worksheet: Teacher's interventions

Allow pupils time to understand and engage with the problem Discourage pupils from rushing in too quickly or from asking you to help too soon.	• *Take you time, don't rush.* • *What do you know?* • *What are you trying to do?* • *What is fixed? What can be changed?* • *Don't ask for help too quickly - try to think it out between you.*
Offer strategic rather than technical hints Avoid simplifying problems for pupils by breaking it down into steps.	• *How could you get started on this problem?* • *What have you tried so far?* • *Can you try a specific example?* • *How can you be systematic here?* • *Can you think of a helpful representation?*
Encourage pupils to consider alternative methods and approaches Encourage pupils to compare their own methods.	• *Is there another way of doing this?* • *Describe your method to the rest of the group* • *Which of these two methods do you prefer and why?*
Encourage explanation Make pupils do the reasoning, and encourage them to explain to one another.	• *Can you explain your method?* • *Can you explain that again differently?* • *Can you put what Sarah just said into your own words?* • *Can you write that down?*
Model thinking and powerful methods When pupils have done all they can, they will learn from being shown a powerful, elegant approach. If this is done at the beginning, however, they will simply imitate the method and not appreciate why it was needed.	• *Now I'm going to try this problem myself, thinking aloud.* • *I might make some mistakes here - try to spot them for me.* • *This is one way of improving the solution.*
	•

3. Structure for IBMT lessons

Workshop activity: Introducing the Theory of Didactical Situations

Aim	Experiencing and introducing phases of the Theory of Didactical Situations (TDS).
Prerequisite	Introducing IBMT
Time	60 minutes
Required material	Ppt and black/whiteboard, copies of the ABBA scenario and cards with names of the TDS phases or a worksheet to be used for participants' comments.
Main issue: Implementation of IBMT requires clearly structured lessons. TDS provides a structure to let students inquire problems and have the teacher try to realize convergence in the learning processes towards intended target knowledge. The names of the phases in TDS provide a terminology to talk about structures of IBMT lessons.	
Task description	The instructor simulates the ABBA scenario with the participants. The participants are asked in retrospect to work in small groups and to describe the target knowledge of the lesson, its structure and its separate phases. They are asked to formulate their findings. The instructor writes the suggested phases on the blackboard and summarizes the findings. Instructor leads a discussion on the number of phases in the lesson – the answers may vary from 2 to 7. Most probably, the phases of TDS emerge. This can be enriched with an activity in which participants are asked to match their phases and terminology with the TDS phases and terminology (through a worksheet) or to organize different teaching and learning activities into phases (through a card organizing game). 

	Next, the instructor gives a (short) interactive lecture on TDS, its phases, and the importance of didactical versus a-didactical phases in a lesson and how institutionalized knowledge is built on personal knowledge of students.
Instructor's actions	The instructor initially simulates ABBA scenario with the participants and poses the task. In reflecting on the scenario participants are challenged to identify phases and match them with TDS terminology. TDS is institutionalized with a short lecture.
Further study	Practical guide, Chapter 3 on TDS. Next suggested activity: Introduction to MERIA scenarios.
Variations based on didactical variables	Use "Race to 20" as an alternative. It is not prepared in the form of a scenario, but it can be used quite well to emphasize the role of the game, milieu and how a winning strategy can be aligned with target knowledge.

Worksheet: Introducing TDS

Which phases in the ABBA scenario are didactical or a-didactical? Label all phases with either: validation, formulation, devolution, action and institutionalisation.

Problem: Look at two-digit numbers, for example 83. For each number look at the difference between the number and its reverse (for 83 this is 38). What do you get if you subtract the smaller one from the bigger one (83-38)? Try again with new numbers. What pattern do you see? Can you explain this?

Phase	Teacher's actions including instructions	Students' actions and reactions
	The teacher states the problem as above, including the example of 83 to make sure the students understand what is meant by "reverse".	Students listen and try to understand the problem.
	Teacher walks round the classroom and registers the students' strategies.	Students try to solve the problem.
	The teacher invites for each significantly different strategy one group to formulate their solution on the blackboard. If students find a solution quickly, the teacher could suggest they try the same problem for more-digit numbers.	Chosen students present their solution. The other students listen, compare their own work to the presented solution and ask questions. There are two options: (1) the groups just found that the difference is divisible by 9, (2) the students found a justification using algebra.
	In case (1) the teacher could lead a classroom discussion on how to know for sure the hypothesis is true for all 2-digit number. The outcome could be approach at different levels of abstraction. Teacher formulates the validation of the students' hypothesis.	Students are engaged in the discussion and ask questions to clarify their understanding of different approaches or steps of the proof.
	The teacher explains how the step $9a - 9b = 9(a - b)$ or $9 \cdot 8 - 9 \cdot 3 = 9(8 - 3)$ is an instance of a more abstract mathematical law $n(a + b) = n\,a + n\,b.$ The teacher can show more instances of this law.	Students write down the conclusions.

Introducing TDS: card organizing game

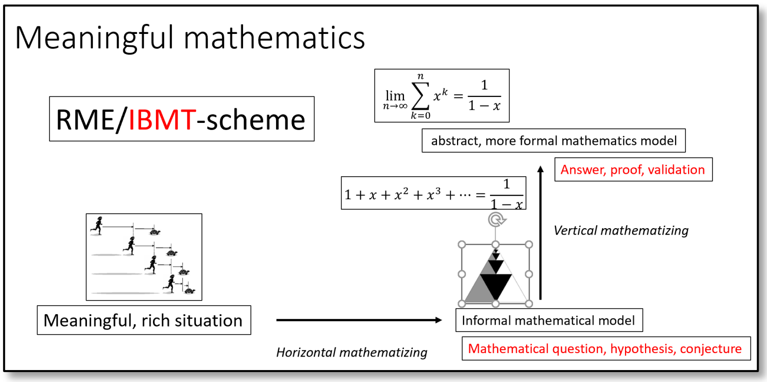
Teacher presents the problem to the students.	Students take the responsibility for solving the problem.	
Teacher checks did the students understand the problem.	Students explore different strategies to solve the problem.	Teacher circulates around the students and observes their work.
If none of the groups advances, teacher opens a discussion and poses a simpler problem.	Students formulate hypotheses.	Students calculate using concrete numbers.
Teacher organizes the presentation of solutions.	Students present their conclusions.	Students write down the conclusions.
Students validate and compare their solutions to others'.	Students (with or without the help of a teacher) prove their statements.	Students discuss the proof.
Teacher connects students' solutions to the target knowledge.	Teacher poses additional questions to check students understanding.	

Workshop activity: Introduction to MERIA scenarios

Aim	To introduce MERIA scenarios. Workshop participants see how ideas from the Theory of Didactical Situation (TDS) can be supported by a concrete template for lesson plans. The participants are introduced to the template through studying MERIA scenarios. Meanwhile they have the opportunity to be inspired to use the MERIA scenarios in their own practice.
Prerequisite	Introducing TDS
Time	60 minutes
Required material	PowerPoint and black/white board or posters for the formulation phase, and a set of scenarios
Main issue: It is time to see MERIA theoretical framework in action. By inspecting the concrete scenarios participants get an idea what IBMT-lessons MERIA proposes.	
Task description	Participants study and report on the MERIA-scenarios.
Instructor's actions	The instructor gives a short introduction to the MERIA scenarios, specifying the target knowledge. Participants form groups of two or three, based on joint interest for a scenario (the target knowledge may be part of the curriculum they teach). The scenarios are divided among the groups. The instructor tries to ensure that all scenarios are explored. Participants study the scenarios in their group. First, work on and analyse the problem. Next, explore the scenario. The participants try to answer the following questions: Do you agree that each phase is needed? Are the activities clearly described? Do you still miss something in case you want to try this out in your classroom? Some groups give a short presentation on a scenario, giving their opinion, criticism, improvements and voicing their expectations of it.
Further study	Practical guide, Chapter 3 on TDS. Next suggested activity: Preparing for workshop homework.

4. Inquiry as part of learning trajectories

Workshop activity: Emergent models in Realistic Mathematics Education

Aim	Learning to identify what informal student models and more formal mathematical models can emerge from working on inquiry based (MERIA) tasks.
Prerequisite	Results of inquiry
Time	60 minutes
Required material	PowerPoint Presentation Online textbook chapters introducing the derivative, e.g.: - Croatia: https://element.hr/artikli/file/1976 - The Netherlands: https://tinyurl.com/afgeleide - Slovenia: http://www.e-um.si/ 4. Letnik: Odvod
Main issue: Knowledge building from IBMT develops in a way different from traditional teaching. Realistic Mathematics Education (RME) provides participants with a supportive theoretical framework to describe this process. By comparing an IBMT task to a traditional textbook (both introducing the derivative) using RME-terminology like *emergent models*, *horizontal* and *vertical mathematization*, participant will see how both approaches reach the target knowledge in different ways. The central issue is that through the IBMT-scenario the target knowledge becomes more meaningful to the student.	
Task description	Part 1: A little lecture using a PowerPoint presentation including slide below (15 minutes)  Part 2: Comparing a mathematics textbook sequence introducing the derivative with the MERIA Slide task. (30 – 45 minutes): - Participants solve the task by constructing a tangent line at a point for a chosen curve. - Participants identify opportunities for mathematization in the Slide scenario (milieu) and the potential of (informal) models, representations, language, etc. developed by

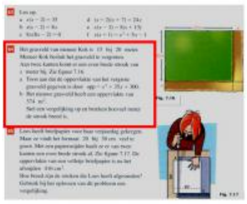
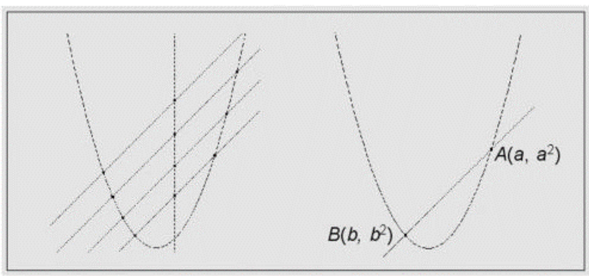
	students and how to reach a taken-as-shared understanding of a mathematical concept or skill (institutionalization). The participants describe as many thoughts/steps/ideas they imagine students will have while working on the problem independently. Participants should come up with: choice of coordinate system, choice of equation/function for the curve, equation for the line, decision of what is fixed and what not in the picture, introduction of parameters, issue of how to design (see approaches in module), issue of how to decide when a solution is good (see approaches in the module). - The instructor points out that students' outcomes will be informal, incomplete and messy. But students will have partial results of their mathematization process - informal student models (as discussed in the lecture), which will be building blocks for (further) vertical mathematization. - Participants discuss how to go from the informal student models to the target knowledge: slope of a curve. - Participants compare this approach with the approach taken in a sample textbook to reach the target knowledge. They describe what students will experience while working with the textbook and compare to the experience they might have with the slide task. - Describe how you would integrate the slide scenario with the textbook. Describe where the experience of the slide scenario can contribute to working in the textbook. Important answer: students can attribute meaning to the more formal notion of derivative by referring to their self-constructed informal models.
Instructor's actions	Giving the lecture. Instructing the participants to solve the slide task using the tangent line. Guiding and discussing this activity. Instructing the participants to identify opportunities for mathematization in the slide task. Discussing with the participants the transition from informal students' models to the more formal models of the target knowledge. Instructing, guiding and discussing the comparison-task.
Further study	Practical guide, Chapter 4 on RME MERIA scenario and module "Slide"

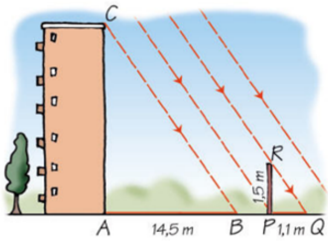
Workshop Activity: The role of contexts in RME

<table>
<tr><td>Aim</td><td>Investigating the role of context in mathematics tasks, in particular on potential from an RME perspective.</td></tr>
<tr><td>Prerequisite</td><td>Basic idea of didactical phenomenology in RME, knowledge of what “realistic” means in RME.</td></tr>
<tr><td>Time</td><td>30 minutes</td></tr>
<tr><td>Required material</td><td>Worksheet with sample tasks</td></tr>
<tr><td colspan="2">Main issue: RME-based tasks have rich, realistic contexts that beg to be mathematized using the target knowledge mathematics. For a novice RME-designer it can be difficult to decide whether a context has these properties properly. And even within a suitable context it can be challenging to discover the full potential for mathematization. This activity aims at promoting these abilities in participants.</td></tr>
<tr><td>Task description</td><td>Watch the video
https://www.youtube.com/watch?v=ujqs2I9Kljo

Participants are asked to engage in a think-pair-share activity: think for 5 minutes, share with the person next to you for 5 minutes, share with everybody for 10 minutes. Questions:
- What RME-principles are important for the choice of a context for a task?
- What is a didactical phenomenology?

Next, participants explore the role of the context in the sample tasks from an RME perspective. Study the examples and answer the questions:
What mathematizations (horizontal/vertical) does the context/situation provoke?
What informal mathematical models can be expected from the students?
Is context important? Can it be omitted? Can it be made important?
Can the context be used in a better/other way?
Can students use common sense to address the context mathematically?
Does the context make sense? Is it used in a sensible way?
Does the context make the mathematics more relevant?</td></tr>
<tr><td>Instructor’s actions</td><td>Present and explain the tasks, including the possible questions. Instruct for the think-pair-share form. Possibly, discuss Task D as an example.</td></tr>
<tr><td>Further study</td><td>Practical guide, Chapter 4 on RME</td></tr>
<tr><td>List of additional materials</td><td>Table of sample tasks with discussion, PowerPoint
In the video http://www.cieaem.org/?q=node/40 Paul Drijvers discusses some of the sample tasks from an RME perspective.</td></tr>
</table>

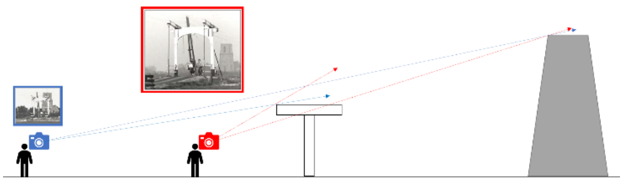
Discussion	Sample task
The context in this exercise does not make sense. How does Mr. Jones know the new area is 374 m²? Did he measure the new dimensions? But then he knows the width of the strip as well! In this situation people would decide or be told: extend your lawn by this many meters on each side. Moreover, the horizontal mathematization has been performed by introducing x and, more generally, by using ideas from algebra.	**Task A**  The lawn in Mr. Jones's garden measures 15 by 20 meters. Mr. Jones decides to extend the lawn. To two sides he adds a strip of equal width of x meters. See Figure 7.16. a. Show that the area of the enlarged lawn is represented by $Area = x^2+35x+300$ b. The new lawn has an area of 374 m². Set up an equation and calculate the width of the strip.
The subject makes sense: describing the (physics) process of a melting ice cube. Unfortunately, the process has been oversimplified. Everyone knows cubes don't melt linear in each dimension. So, students can not really apply their common sense to this situation and actually don't need it. The mathematics does not emerge from the context. In the end: the formula is given, and the student can ignore the context (except for the conclusion that $t \geq 0$): volumes are positive also in mathematics.	**Task B** An ice cube with edges of 30 mm long starts to melt down slowly. Every minute, the edges get 1.5 mm shorter. The volume of the ice cube is described by the formula $V = (30-1{,}5\,t)^3$, where V stands for the volume in mm³ and t for the time in minutes. a. Calculate the volume of the ice cube when t=0. b. What are meaningful values for t? And for V? c. Plot and sketch that part of the graph for which the variables are meaningful. d. Trace the graph with the cursor and investigate after how many minutes the volume is less than 10 000 mm³. Provide your answer with a precision of one decimal.
This task seems not to have a context, but it does have one. The mathematical situation of the "puzzle" forms the context. RME includes this in its notion of contexts. All the more if this situation is modeled in, for example, GeoGebra, so students can have a *hands-on* experience with in. The mathematical knowledge addressed here is: slope (and a bit of algebra). The problem can only be solved by	**Task C**  A parabola is intersected by a straight line. The line is moved upwards. The midpoint of the intersection points seems to move over a vertical line. Is this really the case?

addressing the slope of the diagonal lines: $\frac{a^2-b^2}{a-b} = a+b$ is constant is twice the average of the x-coordinates. Moreover, this lovely little research can be expanded by the teacher to introduce the slope of a curve: $b \to a$ leads to a tangent line with slope $2a$.	
This exercise has a functional context: one could measure the height of a flat this way. Does one ever do this? No, except perhaps as a school practicum (take your class outside and measure the height of the school). Generally, it is not necessary to know the height of buildings surrounding you very precisely. If you need precise measurements of your house, you find the architectural drawings. In other cases, you might estimate the height using different means, for example: number of floors times 3 meter. The exercise does all the horizontal mathematization for the students. Better would have been, for example, “design a way to use the shadow of the building (and a stick) to measure the height of the flat”. In that way the situation does beg to be mathematized by using similarity.	Task D Nienke wants to know the height of a flat. On a sunny day she measures the length of the shadow AB to be 14,5 meters. A 1,5 meter stick has a shadow of length 1,1 meter. (a) Fill in $\Delta ABC \sim \Delta$ ..., because... and ... (b) Make a ratio table and fill in the data (c) Compute AC in meters up to one decimal. 

<table>
<tr>
<td>Of course, the answer to the last question of the task is likely to be “no”, but it is important for learners to ask this question and realise there is a need for a new function. It is also good to dismiss the wrong functions.
The role of the context is to provide the necessity for a new concept, namely logarithmic functions (as the inverse to exponential functions).</td>
<td>Task E
Assumed: no pre-knowledge of logarithms.

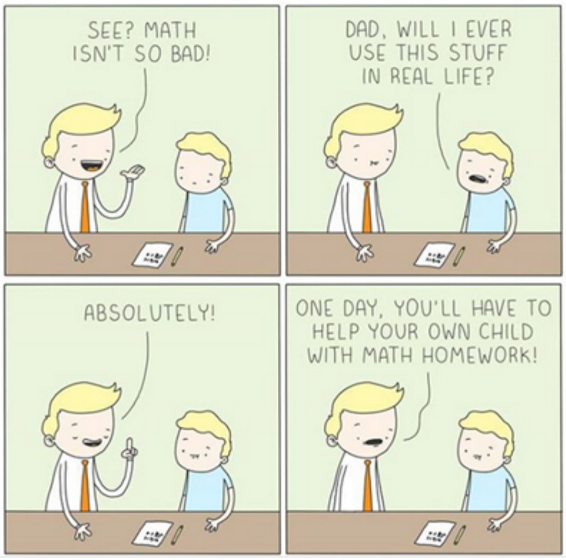
Rogier puts 100 euro’s in the bank. The interest rate is 2%. Fill in the table.
<table>
<tr><td>Amount (A)</td><td>100</td><td>≈ 108,24</td><td>≈ 129,36</td><td>≈ 199,99</td><td>≈ 507,24</td></tr>
<tr><td>Years passed (t)</td><td>0</td><td></td><td></td><td></td><td></td></tr>
</table>

Do you know a function to compute t from A?</td>
</tr>
<tr>
<td>Dutch school children ride their bikes everywhere, in particular to school. Surely, they will have seen bridges and towers like this in relative positions. With their smartphones they take photos (and edit them) daily. Moreover, everyone has an innate ability to imagine scenes from different perspectives. So, this situation is realistic in many ways. And now they are required to think about it mathematically. They will have to introduce notions like viewpoints, projections, vision lines and scaling to discuss the situation, which is the goal of the task.
The Figure on the right summarizes some of the mathematical aspects of the problem. The photos are depicted in a more correct relative scaling.</td>
<td>Task F
Below you see two photos of the same beautiful Dutch landscape with a tower and a bridge from different viewpoints. Which is higher: the tower or the bridge?

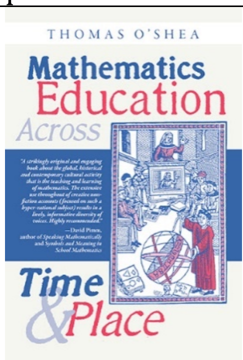
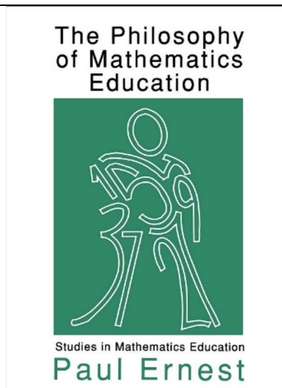
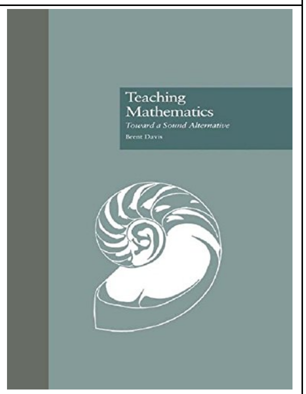

</td>
</tr>
</table>

Workshop activity: Mathematics education for RIA mathematics

<table>
<tr><td>Aim</td><td>To become aware of goals of mathematics education, the options in education to have students experience how it is Relevant, Interesting and Applicable, how mathematics relates to their daily life. How do the MERIA scenarios support this goal?</td></tr>
<tr><td>Prerequisite</td><td>The MERIA scenarios (ideally experiences with implementation)</td></tr>
<tr><td>Time</td><td>30 minutes</td></tr>
<tr><td>Required material</td><td>Pen and paper</td></tr>
<tr><td colspan="2">Main issue: The daily practice of teachers is often dominated by covering the textbook within the time available and preparing their students for final examinations. In this workshop activity we go beyond these limitations of the educational system. The goals of mathematics education are challenged, and participants are invited to think beyond daily practices. We expect participants to become aware of goals related to relevance, interest and applications (RIA). Furthermore, they are invited to read more about why we teach mathematics and to what extent the MERIA scenarios connect to these RIA goals of mathematics education.</td></tr>
<tr><td>Tasks</td><td>Ask the participants why they teach mathematics. This question can be introduced with the following comic:



Invite the participants to think about goals of mathematics education broader than preparing for final exams.
The instructor writes down the answers on the blackboard, at the same time trying to create categories that can be labelled with Relevant, Interesting and Applicable.
With relevant we refer to keywords like important, valuable, universal, overarching, logical, natural, empowering, and influential. Interesting refers to motivating, inclusive, encouraging, enjoyable, fun, engaging, activating, collaborative, and experiential.</td></tr>
</table>

	Applicable means useful, meaningful, purposeful, omnipresent, operational, understandable, and connected to reality. Limit each category to 4-7 words. Instructor can stop at a certain word and ask what differences are meant by two closely related words (e.g. challenge whether mathematics should be fun or interesting). The idea is not to have as many words possible, but to reflect on the meaning of these words. Next, ask the participants to relate these goals to their daily practice (could be organized in a think-pair-share activity). Finally, ask to what extent they recognize these elements in (experiences with) the MERIA scenarios.
Instructor's actions	Build on participants' contributions. Create an atmosphere in which goals of education go beyond preparing for the test. Make the RIA notions richer and invite participants to recognize clues how they can be encouraged in educational practices.
Further study	  

5. Experiencing MERIA scenario's

Workshop Activity: Introducing MERIA modules

Aim	To Introduce MERIA modules. Workshop participants see how MERIA modules support them in preparing for lessons based on MERIA scenarios. Participants should feel more confident in implementing scenarios in their classroom.
Prerequisite	Introduction to MERIA scenarios
Time	60 minutes
Required material	PowerPoint and worksheets
Main issue: Teachers are confronted with some problems in implementing MERIA scenarios. MERIA modules provide additional materials and help them tackle some of the problems.	
Task description	Workshop participants in the groups study the parts of the MERIA module and discuss the most important parts with others.
Instructor's actions	The instructor gives a short introduction to the MERIA modules, describing the structure of the MERIA module. Participants will study the modules using the Braking distance module, so the teacher briefly presents this scenario. Participants form seven expert groups. Each expert group receives its own worksheet with one part of the module. In an expert group, participants study the part of the module within their group and highlight the most important part. The participants form new groups consisting of one representative of each expert group. In the new group, each member presents to the other members part of the module of his expert group. After listening to all the presentations, they collectively discuss how parts of the module can help them perform scenarios. The conclusions are shared in the plenary discussion.
Further study	MERIA teaching and learning modules

Workshop activity: Preparing for workshop homework

Aim	Preparing for homework: what to prepare for implementing a scenario, how to reflect on implementation
Prerequisite	Introduction to MERIA scenarios
Time	30 minutes
Required material	Ppt, worksheets and black/white board
Main issue: IBMT requires inquiry teaching skills. Instruction for this behaviour can be found in the scenarios. However, to perform these scenarios in a successful implementation a good preparation is needed. This activity supports teachers to think in advance of the devolution, formulation and observation of students, the classroom atmosphere, and their own actions.	
Task description	Participants choose what scenario they will implement in their practice and prepare a presentation (ppt, poster, handout, black/white board) for the devolution of that scenario. Participants decide on the way they are going to organize formulation (divide the black/white board, use posters, or share screen). Participants, in pairs from the same school, prepare observation forms, for instance with tables to distinguish between what you see and why you think it is remarkable, see the general example table *Worksheet: organizing and selecting students' approaches* or the more detailed observation form for the slide scenario *Worksheet: Example observation form for MERIA-scenario "Slide"* and give feedback on each other's preparations (this can be done at their school). The participants are asked to collect some student materials during the experiment. Furthermore, they are asked to fill out a reflection form with questions: *What did you try out? What was the most surprising? What was most positive? Who has learnt what and how do you know?* See *Worksheet: reflection form MERIA scenario*. Participants are encouraged to first try out the ABBA scenario (or another short one) to prepare their students for this kind of activity. Consequently, after the short activity they need to talk with their students about their inquiry to make clear what is expected from them.

Instructor's actions	Ask participants to select a scenario and clearly describe that this activity supports them to implement the scenario. Give the first task and let the participants work on devolution and formulation on their own for 15 minutes. The results are not to be discussed. Ask the participants to prepare the tables for observation by listing the strategies. Participants may ask questions about their scenario. Present the next activity of filling out the third column of the scenario with expectations of the hypothetical trajectory. Show an example observation form and a sample of observations and discuss what is expected from the observer of the lesson: not short answers. Ask the participants which of the observations they find valuable and why, and which observations are rather limited and create the need for more information about what happened in the lesson. A discussion is expected to prepare them to better understand what to observe and note during the test lessons. Explain that it can be very challenging to observe the students' approaches and evaluate if and how they contribute towards the learning goal; see the activity "Results of inquiry". Finally, invite the participants to report by answering the questions.
Further study	Practical guide.

Worksheet: Example observation form for MERIA-scenario "Slide"

Observer: ____________________

Design strategies

☐ Translating the curve	☐ Finding equations in GeoGebra
☐ Translating the line	☐ Determine slope with two points
☐ Vary slope of the line	☐
☐ Vary parameters curve	☐
☐ Use one point on the curve	☐
☐ Use two points on the curve	☐

General remarks about the strategy (e.g. "students first make drawings", "students fix the curve and vary the line")

Solution/design attempts

Line	Curve

Interesting substantive quotes and use of concepts by students (e.g. about tangency, tangent lines, slope, slope coefficients, ...)

Validation strategies

☐ Precise drawings	☐ Focus on intersection points
☐ Digital drawings	☐ Computing intersection points
☐ Zooming in (digitally)	☐ Using the discriminant (in case of quadratic equations) to determine the number of solutions
☐	☐ Numerically determine the average slope of the curve
☐	☐
☐	

Worksheet: Filled in sample example observation form for Slide scenario

Observer: M. Doorman

Design strategies

Translating the curve	☐ Finding equations in GeoGebra
☑ Translating the line	☐ Determine slope with two points
☑ Vary slope of the line	☐ ...
☐ Vary parameters curve	☐ ...
☑ Use one point on the curve	☐ ...
☐ Use two points on the curve	☐ ...

General remarks about strategy (e.g. "students first make drawings", "Students fix the curve and vary the line")

13:15 Start lesson. Rogier introduces the problem. Sketches a slide, smooth connection, equations. Hand out worksheets. Gives a working schedule; sets students to work

13:20 Start!
Students hesitate on how to begin. They are supported by Rogier's reminder of y=ax+b: "that's for the straight line" "Now we need to make a curve" "that could be with x squared".
Students types on his phone: "formula for curve"

"How high is a ski-jump?"
"140 meter" "We'll go for 100"

Attempt 1: y=4x+100. Put in Graphical calculator. "Then he goes 4 meter down per meter. That's too much". "it has to be -4x".
Students perform ZoomFit on GC. "I don't think this looks so steep" "Now we need a curved bit attached to it".

Students are very pleased with their parabola. (One student struggles with a minus sign)

"Difficult to move the parabola to the right!"

One student knows how to move the line to the left instead.

Solution/design attempts

Line	Curve
$y = 4x + 100$	
$y = -4x + 100$	$y = -4x^2 + 1$
$y = -4x + 50$	
$y = -6x - 59$	$y = 0.15x^2$

Interesting substantive quotes and use of concepts by students (e.g. about tangency, tangent lines, slope, slope coefficients, ...)

"You would like to have less angle (between the curve and line)"
"Look at the intersection points"

Validation strategies

☑ Precise drawings	☑ Focus on intersection points
☑ Digital drawings	☐ Computing intersection points
☐ Zooming in (digitally)	☐ Using the discriminant (in case of quadratic equations) to determine the number of solutions
☐ ...	☐ Numerically determine the average slope of the curve
☐ ...	☐ ...
☐ ...	

Worksheet: Reflection form MERIA scenario

<table>
<tr><td>Name</td><td colspan="3"></td></tr>
<tr><td>Subject</td><td>Mathematics</td><td>Class</td><td></td></tr>
<tr><td rowspan="3">Which scenario was used (short description)?</td><td>Book, paragraph</td><td colspan="2"></td></tr>
<tr><td colspan="2">Replacement for book or extra material?</td><td></td></tr>
<tr><td colspan="3"></td></tr>
<tr><td>Did you adapt the scenario, or what did you include specifically in your lesson to stimulate IBMT?</td><td colspan="3"></td></tr>
<tr><td>Why do you think that this/your scenario will stimulate IBMT?</td><td colspan="3"></td></tr>
<tr><td>Experiences during the lesson: what was student behaviour like and what was the yield (different than normal)</td><td colspan="3"></td></tr>
<tr><td>What was the most surprising? What was most positive? Who has learnt what and how do you know?</td><td colspan="3"></td></tr>
<tr><td>Will you do this activity or a similar one more often?</td><td colspan="3"></td></tr>
<tr><td>What else would you like to mention?</td><td colspan="3"></td></tr>
</table>

Worksheet: Organizing and selecting students' approaches

Group	Approach	Reason(s) for selecting for formulation

Workshop activity: Reflecting on implementation of MERIA scenarios

Aim	Participants report on their implementation of a MERIA scenario and share similarities and differences
Prerequisite	Preparing for workshop homework
Time	60-90 minutes
Required material	Ppt, participants' written reports, large papers
Main issue: One of the key elements of the MERIA professional development course are the experience of the MERIA scenarios in own classroom and the reflection after the implementation. As participants experiment with a new way of teaching it is useful to receive feedback and to rethink their decisions. This activity helps participants to make their decisions conscious and compare their actions and the actions of their students with experiences in other schools.	
Task description	Following the implementation of a MERIA scenario in their classroom, participants should discuss their experience with colleagues from their school (who have ideally attended the lesson) and write a simple report, based on the observation and reflection forms and preferably, if available, student work. The report is sent to the instructor between the meetings. The instructor organizes a discussion based on the reports. The discussion covers various phases and aspects of the lesson focusing on the expectations, obstacles, issues and possible improvements. The instructor can also organize a discussion based on the observation form and the student work. This can be compared to the way the institutionalization was realized. Has the potential of the informal student approaches been used in the institutionalization? Does more formal mathematics emerge from the students' productions? After the discussion, participants form groups and write their conclusions in the form of a poster. Groups may be formed based on the scenario they have implemented. The posters are presented in plenum.
Instructor's actions	Instructor gathers participants' reports and organizes the discussion. When necessary, the instructor focuses the discussion on a single aspect or poses additional questions. The participants are asked to write their conclusions in the form of a poster and present to others.
Further study	Practical guide on IBMT.

Workshop activity: Including previous experiences

Aim	Participants learn from other participants' experience how to work with scenarios
Prerequisite	Some teachers with experience using the MERIA scenarios
Time	30-45 minutes
Required material	Ppt and black/white board, materials from testing of scenarios, reflections
Main issue: A teacher that encounters IBMT for the first time may be confused and insecure about the process and the teacher's role in it. It is valuable to learn from participants who already have experiences in MERIA scenarios.	
Task description	Participants with experience provide reflection on these experiences (25 minutes): What was the scenario that was implemented? What was the most positive and what was the main challenge? How did students react? What have I, as a teacher, learned in the process of using scenarios? What would I, as a teacher, suggest to new teachers? What would I, as a teacher, like to share with others? What am I, as a teacher, proud of? Then follows a discussion about the presented reflections and possible questions from participants (20 minutes).
Instructor's actions	The instructor choses 2-4 participants with experience for the presentations according to the scenarios and preferably different type of schools. The instructor organizes the reflections and leads the discussion.
Further study	Practical guide on IBMT.

Workshop activity: Sustainable innovation

Aim	Teachers explore ways of sustaining IBMT in their schools
Prerequisite	Introduction to MERIA scenarios
Time	15-30 minutes
Required material	Ppt, black/white board
Main issue: For teachers who are involved in MERIA workshops it is very valuable to have support from other mathematics teachers in their school.	
Task description	The activity consists of a brainstorm-session in small groups (4 - 6) on this issue. Participants from the same schools discuss in the same group. Questions for the brainstorm-session are: What can you as a teacher with experience do to promote new ways of teaching in your school? What kind of support do you need from MERIA project team? How do you see the results of MERIA project as sustainable?
Instructor's actions	Instructor presents questions (ppt) and organizes the brainstorm-session. The participants share their ideas and answers and the instructor records these on the board. The instructor leads a reflective discussion on the results of the session.
Further study	Practical guide on IBMT.