MERIA Scenario "Conflict lines – introduction"

Partitioning of a plane by perpendicular bisectors

Target knowledge	The partitioning of a plane by perpendicular bisectors of pairs of given points.
Broader goals	Construction of a perpendicular bisector. Understanding the characterization of a perpendicular bisector as the collection of points that have equal distance to two given points. Characteristics of bisectors and their points of intersection in triangles and quadrilaterals, and characteristics of points in regions determined by perpendicular bisectors. The ability to operate with the notation d(P,X) < d(P,Y). Inquiry skills: experimenting and drawing systematically to create areas or borders of areas that are determined by (distances to) given points. Presenting findings clearly by making decisions which lines to emphasize. Interdisciplinary skills: students can connect territorial problems or conflicts (geography) to geometrical ways of representing and solving these conflicts. Other problems may be used to discuss application to robot navigation.
Prerequisite mathematical knowledge	Pythagoras and triangle inequality (in particular for the proof).
Grade	Age 15 - 16, grade 9 - 10 (whenever the perpendicular bisector is introduced)
Time	40 minutes, with applet 70 minutes
Required material	Worksheets, paper, ICT and MERIA applet in GeoGebra: https://meria-project.eu/applet/voronoi/voronoi.html Alternative sites: http://alexbeutel.com/webgl/voronoi.html, https://www.desmos.com/calculator/ejatebvup4

Observations from implementation

The context of observations (grade, institution, country, etc.):

Problem:

Given a collection of water wells in a desert. Students are asked to colour areas in the desert in such a way that for each possible point in a coloured area the corresponding well should be the one that is the closest to that point. [1]

[1] The problem and the map of the desert was intoduced in the book Geometry with Applications and Proofs, Voronoi Diagrams by A. Goddijn, M. Kindt, W. Reuter

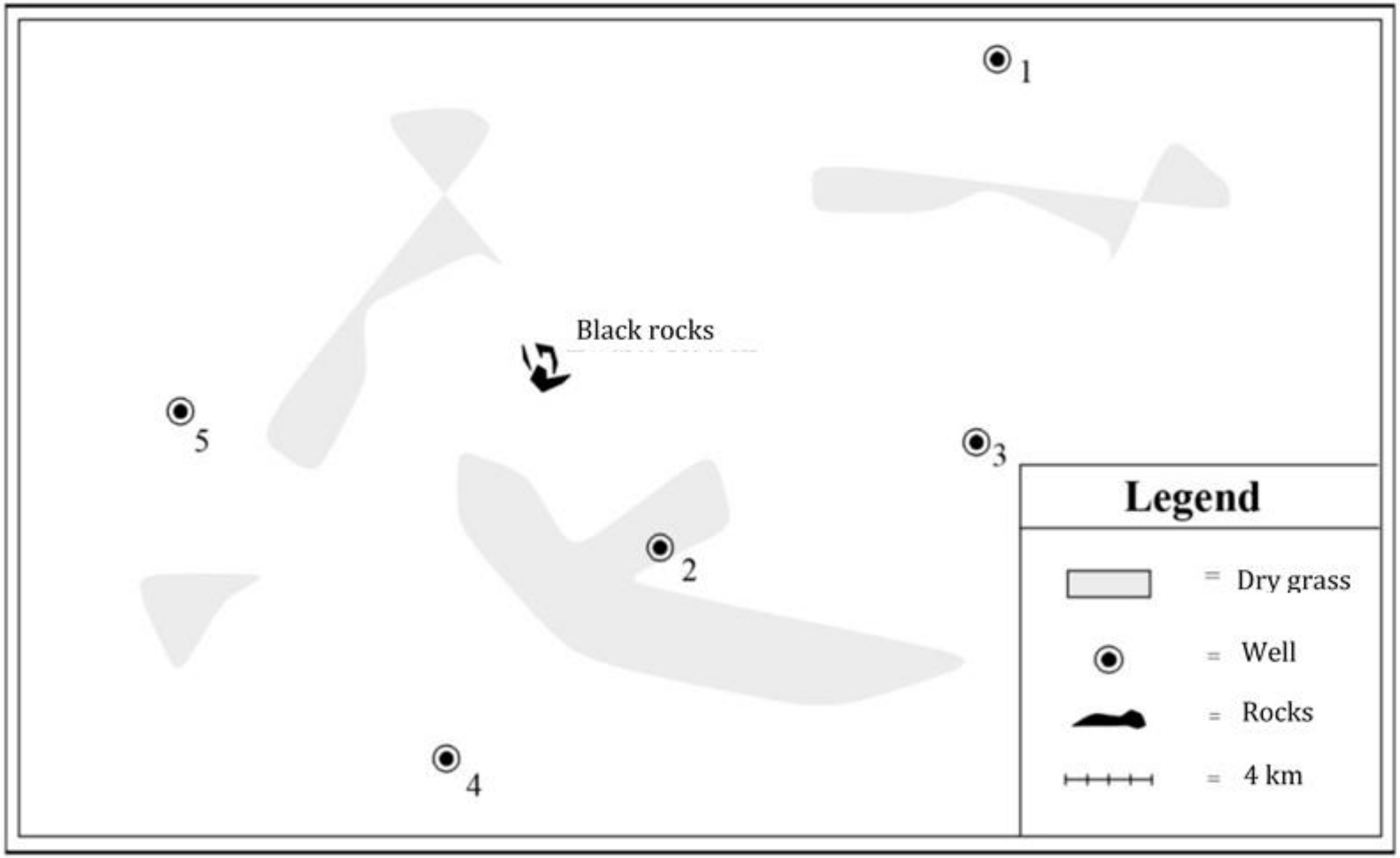
Phase	Teacher's actions incl. instructions	Students' actions and reactions	Observations from implementation
Devolution 1 (didactical) 5 minutes	Introduce the notion of a conflict line in the classroom: Suppose two students (X and Y) have some sweets and you are asked to go for a sweet to the one who is closest to you. The teacher selects the two students and asks: who is closer to student X and who to student Y, and finally let students raise hands that have difficulty in deciding …	Students participate by raising hands and feel being a point in a plane and deciding themselves whether they are closer or not to one of the two given points. Moreover, they see how others decide.	
Institutionalisation (didactical) 2 minutes	The teacher summarizes the main finding: The problem is to identify points with 'same distance' and the challenge is to find some kind of procedure for being sure and precise about the collection of points with that characteristic. Consensus is established on notation (e.g. d(A,C)<d(B,C) for point C being closer to A than to B). This will be elaborated in the next step (students will get the opportunity to work with notation and distance-related reasoning).	Students listen and are able to connect the institutionalized reasoning and notation to their own work.	
Devolution 2 (didactical) 3 minutes	The teacher sets a new problem: Locate yourself somewhere in the desert (provide a worksheet to students). Find the well that is the closest to you. Find all positions from which you would also go to that well. Finally, divide the map into regions around wells, such that for each well all points in the corresponding region are closest to that particular well.	Students listen.	

Action (adidactical) 15 minutes	Teacher circulates in the classroom.	After plotting the position and detecting the closest well, groups start to construct the region with all such points – the closest well paired with others, one by one. To divide into regions students discover that they need some kind of strategy (proof) because soon, with more points, things become complicated.	
Formulation (adidactical) 5 minutes	Teacher circulates in the classroom to identify what different ideas the students recall and use and announces presentations.	Students discuss in groups what they did, what the set of points with the requested property is and how to write it down.	
Validation (didactical and adidactical) 5 minutes	Teacher asks some groups to present what they did so far (if possible, at least a group that uses equidistant circles and a group that started drawing bisectors).	Students present.	
Institutionalisation (didactical) 5 minutes	Teacher highlights the fundamental theorem underlying what they did: d(A,P) = d(B,P) if and only if P is on the perpendicular bisector. Voronoi diagrams are constructed with perpendicular bisectors, so these are the basis of algorithms to construct those diagrams. In addition, definitions of bisectors can be discussed: “the collection of points with equal distance to points A and B”, and “the line through midpoint and perpendicular to AB”. An optional part of the scenario: Can you prove the theorem?	Students understand the introduced notation as it refers to their activity, e.g. d(A,P)=d(B,P) defines a line of points P (so-called “conflict line” for points A and B). d(A,P)<d(B,P) defines a region (so-called “safe region”), and they understand the mathematical problem as it also emerged in their activity.	

	(not all students feel the need for proof and see it as a challenge)		
Devolution 3 (optional) 5 minutes	ICT can be used to draw Voronoi diagrams. The teacher demonstrates what an ICT program can do (after entering two points). Recall/discover there is a line with equidistant points, and the plane is divided into 2 regions. Then continue with 3 points and find/discover there exists an equidistant point to all 3 points. Use for instance: https://meria-project.eu/applet/voronoi/voronoi.html. Revisit your original problem-situations and explore what happens in specific cases. Play around and find a nice surprising pattern with a set of structured points or, for instance, explore what patterns you can get with 4 points moving around.	Students listen and watch the software drawing Voronoi diagrams automatically. They become interested and challenged to use it by themselves and investigate what happens when playing around.	
Action (adidactical) 10 minutes	Teacher circulates in the classroom, challenging students to experiment systematically, only supporting them when they have problems with operating the software. When many have a similar problem, deal with that problem plenary (e.g. note it on a visible spot).	Students construct their original problem in the software, find the solution and compare it with their original drawing. They also explore what happens in other cases with regularly and/or irregularly distributed points.	
Formulation (adidactical) 5 minutes	Teacher asks them to prepare a presentation of their (most surprising) findings and challenges them to find justifications for specific patterns (e.g., when does a 4-point Voronoi diagram have one 4-region point?).	Students prepare two screenshots, one of the solutions of the original problem and one of their nice pattern (and how they constructed it). They try to formulate justifications for their findings using the	

		circle-tool and the formal distance-notation and theorems like Thales, Pythagoras …	
Validation (didactical and adidactical) 5 minutes	The presentations help to validate what happens in these diagrams, to get familiar with the formal notation and to use geometrical reasoning in different partitioning situations.	Students see the connection between the validations and the formulations of their findings.	
Institutionalisation (didactical) 5 minutes	General conclusions of the concept of Voronoi diagrams consisting of perpendicular bisectors, and some illustrative cases and patterns in these diagrams.	Students realize how institutionalized learning goals are connected to their initial explorations in the desert-context and have acquired these learning goals.	

Possible ways for students to realize target knowledge	• Some students will start sketching lines between the given points that have more or less curved pieces and no clear intersection-points where three (or four) lines meet. • Some students will draw circles or divide the areas with curved lines. These students need to realize that curved lines are impossible and that drawing circles is helpful for finding points with the same distance to a well or center, but not for finding borders (although they can be used for that). • Some students might immediately know what to do and start drawing bisectors. For them, the crucial point is to discuss what happens in areas where bisectors meet. Do they meet at one meet point?

Worksheet	 1 Black rocks 5 3 2 4 Legend = Dry grass = Well = Rocks = 4 km