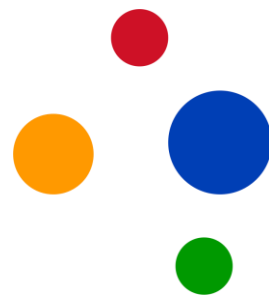




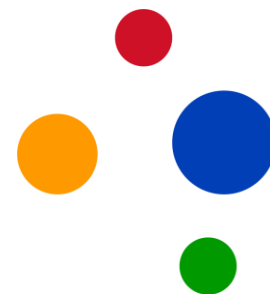
Mathematics Education -
Relevant, Interesting and Applicable

MERIA TEMPLATE FOR SCENARIOS AND MODULES





(this page is intentionally left blank)



MERIA TEMPLATE FOR SCENARIOS AND MODULES

EDITOR IN CHIEF

Carl Winsløw

TEXT WRITTEN BY

Britta Jessen, Carl Winsløw

REVIEWS, EDITING AND PROOF READING

Sanja Antoliš, Jeanette Axelsen, Matija Bašić, Rogier Bos, Kristijan Cafuta, Aneta Copic, Gregor Dolinar, Michiel Doorman, Željka Milin Šipuš, Selena Praprotnik, Sonja Rajh, Mateja Sirknik, Mojca Suban, Eva Špalj, Carl Winsløw, Petra Žugec, Vesna Županović

DESIGN AND VISUALS

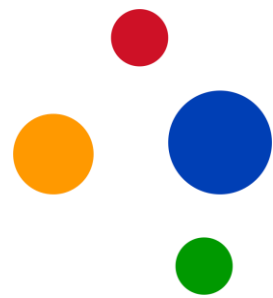
Irina Rinkovec

Project MERIA, February 2018.

www.meria-project.eu

This document is protected under a creative commons license.

The content of this document reflects only the authors' view. The European Commission is not liable for any use that may be made of the information contained therein.



(this page is intentionally left blank)

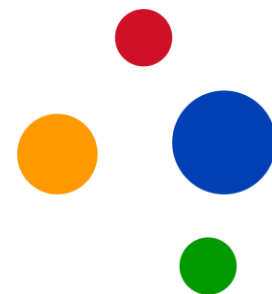
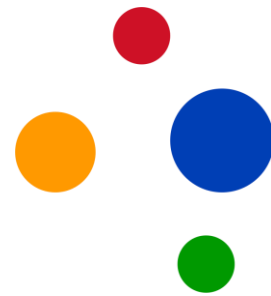


Table of Contents

Introduction	2
1. Template for MERIA scenario	3
2. Template for MERIA module	4
The scenario	4
Explanation of materials	4
Variations based on didactic variables	4
Observations from practice	4
Evaluation tools.....	4
Rationale and RME perspectives on the scenario.....	4
3. MERIA scenario and module: Area enlargement.....	5
The teaching scenario	5
Explanation of materials.....	9
Variations based on didactic variables	10
Observations from practice	11
Evaluation tools.....	12
Suggestion for further problems regarding area and enlargement.....	12
Rationale and RME perspectives on the scenario.....	13

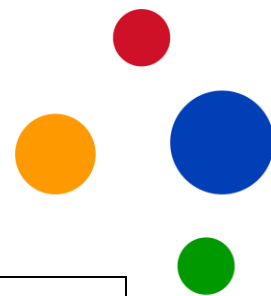


Introduction

One of the main results of the project MERIA will be a set of new showcase teaching scenarios and modules, which will be based on the theoretical background presented in the *MERIA Practical guide to inquiry-based mathematics teaching*.

A *scenario* describes a didactical situation and corresponding teaching methods. It describes the goals of a teaching lesson in terms of curriculum area, specific target knowledge and competency and provides a clear structure for the lesson based on the Theory of Didactical Situations. A *module* also contains written and digital materials accompanying the teaching scenario, such as student assignments or digital worksheets. It contains the explicit rationale for the choice of the problem(s) and the teaching methods, a perspective from Realistic Mathematics Education, experiences and results from controlled implementations, including potential gains and pitfalls for students with specific prerequisites.

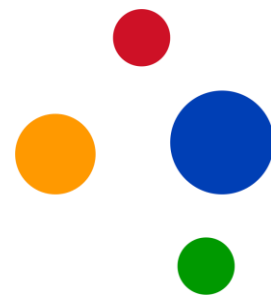
This booklet contains the templates describing the structure of a scenario and a module, as well as one example called *Area enlargement*. This exemplary scenario has been trialled in associated schools in all four partnering countries (information about the schools is available on the project's webpage) and the information in the module is based on the data and feedback gathered by the project team from these trials.



1. Template for MERIA scenario

Target knowledge	A precise mathematical formulation of the goal.		
Broader goals	Broader achievements such as competences, possible applications, reasoning etc.		
Prerequisite mathematical knowledge	Precise formulation of what mathematical knowledge, skills and competences the students are expected to possess (before engaging with this situation).		
Grade	Grade number and age of students.		
Time	Estimated time and number of lessons. (45-60 minutes entities).		
Required material	All sorts of needed artefacts.		
Problem: The exact formulation of the main problem, which the teacher devolves to the students (possibly after some preparatory activity).			
	Teacher's actions incl. instructions	Students' actions and reactions	Observations from implementation
Devolution (didactical) Time estimate			
Action (adidactical) Time estimate			
Formulation (didactical/adidactical) Time estimate			
Validation (didactical/adidactical) Time estimate			
Institutionalisation (didactical) Time estimate			
Possible ways for students to realize target knowledge	<ul style="list-style-type: none"> - Be mathematically explicit about the strategies that students might follow. Remember to emphasize when a strategy can split into a scenario with ICT or without ICT using only pen and paper, as well as if the strategy requires to look at special cases. 		
Further study	<ul style="list-style-type: none"> - What are possible applications / generalization of the notion or the concept studied? 		
List of additional materials	<ul style="list-style-type: none"> - Students' productions (snapshots of boards, reports, assignments, posters etc.) - Formulations of students' assignments, reports or other productions required from students based on the lesson - Table for recording students' strategies - Video 		

Note: the phases of devolution and institutionalization can be repeated, but should not be repeated too often in one situation.



2. Template for MERIA module

The scenario

A short description of the scenario is written in table format. Remember to indicate time, instructions and possible different student strategies together with the goals of the scenario. Column for observations will be omitted in the module.

Explanation of materials

In this section a short explanation on how to use additional materials should be provided presenting each possible material or resource (hand out, (electronic) worksheets, dice, building blocks and other artefacts) and suggesting how to use it in a certain phase. Some of the resources might be for the teacher only.

Variations based on didactic variables

In this section, possible variations are described together with recommendations regarding what could be changed and what should not be changed. The variations can be based on changes of the milieu (changing the use of ICT, adding or removing mathematical objects or artefacts), the didactic time (a particular phase being shorter/longer) or the organization of the class work individually, in pairs or in groups. Further, the impact and consequences of variations should be discussed based on the context they are to be realized in – and draw on experiences from the testing of the scenario.

Observations from practice

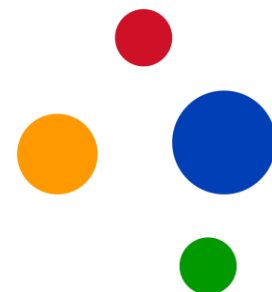
Here the main observations and reflections from observing the test implementations of the scenario should be shared. This means a description of less successful strategies and suggestions on when and how to intervene, description of strategies with different level of success supported by data (photo of students' work, assignments etc.)

Evaluation tools

It can be an advantage to deliver an evaluation tool in the form of an immediate task (or more simple tasks) the students are supposed to be able to solve if they have reached the learning goal of the scenario. Also, to provide suggestions for further problems regarding the target knowledge. The possible strategies suggested in the scenario can be used as a "checking tool" during teaching. If the teacher creates a sheet with the list of strategies, (s)he can note which groups follow which strategy and whether any important ones are left out. This can be addressed during the scenario, but also used after the lesson for reflection.

Rationale and RME perspectives on the scenario

An elaborated presentation of how the target knowledge can be reached with the scenario. Further, RME elements can be addressed, such as: choice of mathematical target knowledge, relevance and applicability, inquiry skills and potential for additional lessons (providing a storyline for a whole chapter or presenting "a bigger picture").



3. MERIA scenario and module: Area enlargement

The teaching scenario

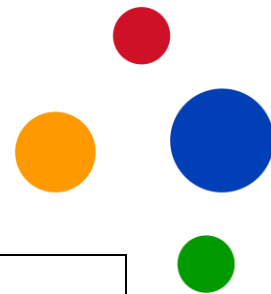
Target knowledge	Whenever all side lengths of a polygon are enlarged by a certain factor k , the area of the polygon is enlarged by the factor k^2 .
Broader goals	<p>Autonomous algebraic and geometric reasoning, formulation of general statements and proofs based on formulas of circumferences and areas of different shapes, possibly including the sine function as well as additivity of area when cutting the polygon in parts. The notion of similar polygons.</p> <p>If students are used to work with ICT: to generate hypotheses in a graphical environment and use it as an outset of a proof.</p> <p><i>More on the relevance, perspectives and further developments of this scenario on pp. 13-15.</i></p>
Prerequisite mathematical knowledge	<p>Students need to have some knowledge of how to compute area of polygons, including triangles and squares.</p> <p>Also needed: notion of similarity, magnifying polygons by a scale factor.</p>
Grade	Grade 10, students aged 15-16 years
Time	90 minutes, two lessons
Required material	Pen, paper, grid paper, ruler, a mathematics tool which allows to draw and measure polygons (e.g. Geogebra), a device which allows to manipulate pictures (smartphone, pc, or tablet). The use of technology is strictly speaking not needed but greatly enhances the familiarity of the environment for most students.



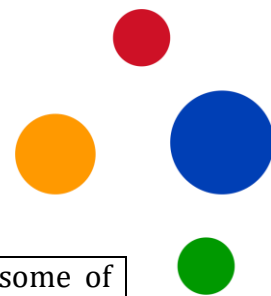
Student presentation from this situation, in Croatia.



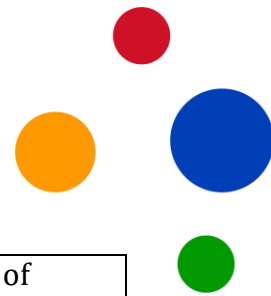
Problem: Look at these two pictures. If you open them on your smartphone or computer, you can easily drag the pictures in order to enlarge them. But what happens with the areas of the pyramid and of the black building when we enlarge the picture?



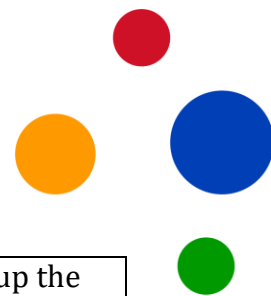
Phases	Teacher's actions incl. instructions	Students' actions
Devolution (didactical) 2 minutes	The teacher starts by asking: What do I need to know in order to find the area of a triangle? Or the area of any other polygon? There is more than one answer and you are welcome to provide several answers. Write down the answer on a piece of paper. You have 2 minutes to do this.	Students accept the task, and possibly ask clarifying questions to make sure they understand their task.
Action and formulation (adidactical) 2 minutes	The teacher walks around the room and identifies what different ideas the students recall and write down.	The students write down formulas such as $A_{square} = l \cdot h,$ $A_{triangle} = \frac{h \cdot b}{2},$ $A = \frac{a \cdot b \cdot \sin(C)}{2}.$ They may also note that the area of any polygon may be calculated using triangulation. Other methods: Counting squares on a grid paper; Computer based methods.
Validation (didactical) 5 minutes	The teacher chooses students to present their writings at the board in order to get all strategies represented. The teacher asks the class to pose questions or comments to the presentations.	The students listen to those presenting, and ask for elaboration, comment or discuss the suggestions at the board.
Institutionalisation (didactical) 2 minutes	The teacher sums up the number of ways to find areas of polygons.	Students listen.
Devolution (didactical) 2 minutes	The students are divided into groups of 3, but should start working individually. They have 15 minutes to prepare their own answer to the problem stated above. Ask if students understand it. The students are provided with (or asked to bring) plane paper, grid paper, scissor, ruler, calculator and computer with relevant ICT.	Students listen and pose clarifying questions if needed. They pick up materials needed, if they want to use paper, ruler etc.



Action (adidactical) 15 minutes	The teacher circulates the classroom to note what strategies students choose. <i>The teacher does not interact except to clarify the problem.</i>	Students start to try some of the strategies in their group. For possible strategies see below.
Formulation (adidactical) 10 minutes	The teacher asks the groups to agree upon one answer to the problem by presenting and discussing their personal ideas. The teacher surveys the group work so (s)he can organise the presentations.	Students give a short presentation of their work and the group refines the presentation of the chosen strategy.
Validation (didactical) 20 minutes	The teacher calls the groups to present one by one, starting with the most practical and vague formulations, and ending up with the most general arguments. The class is encouraged to pose elaborating questions together with the teacher during other groups' presentations.	Students give their best possible presentations, listen and pose elaborating questions if other presentations are unclear to them.
Devolution (didactical) 2 minutes	The teacher asks the groups to explain relations and differences among the answers presented. Which is "most useful" and why?	Students accept the task.
Action/ formulation (adidactical) 15 minutes	(S)he observes the arguments formulated in groups.	Student might build arguments on examples, calculations or algebraic manipulations.
Validation (didactical) 10 minutes	The teacher uses the knowledge about the work of the individual groups to sequence and select different presentations of answers, so all strategies are represented.	Groups present their answers by using the board. Other groups pose clarifying questions or add comments when relevant.
Institutiona- lisation (didactical) 5 minutes	The teacher sums up by emphasizing the different strategies. (S)he formulates how the strategies are related and support each other, though some strategies are preferred in certain cases (e.g. new examples). The teacher formulates the target knowledge in its general form, pointing out how it appears in the different solutions proposed by students.	Students listen and some might take notes.
Total: 90 minutes		



<p>Possible ways for students to realize target knowledge</p>	<ul style="list-style-type: none"> ○ Drawing the polygons on grid paper, counting the number of covered squares in the original and enlarged figures (without explicit use of the concept of scale factor of lengths). ○ Drawing on any piece of paper, measuring the baseline, b, and the height, h, with a ruler in order to calculate the area using $A = \frac{h \cdot b}{2}$ (again without scale factor of side lengths). ○ Experimenting with different scale factors of lengths (2, 3, 0.5 etc.), arriving at hypotheses such as employing a factor 2 leads to areas increased by 4, etc. <ul style="list-style-type: none"> ○ Can be realized algebraically from examples (choose dimension of a triangle) ○ The above strategy can be carried out using ICT ○ It can be realized with grid paper. Draw different shapes, enlarge them and count the number of squares covered ○ It can be realized by drawing triangles on paper, use the ruler to measure the side lengths and calculate the areas ○ The shapes can be drawn using ICT, e.g. Geogebra. Side lengths and areas may be measured using instrumented techniques, depending on the tool. ○ Draw the triangles and cut them out of the paper. If the enlargement is an integer, k, then the small triangle can be fitted k^2 times in the enlarged one. <div style="text-align: center; margin: 20px 0;"> </div> <ul style="list-style-type: none"> ○ Using ICT: draw the polygon, drag it until it is enlarged by a certain amount and ask the program to calculate the areas. For instance, students can use geometric sketchpad for enlarging pictures and observe what happens to the area. ○ Based on experiments as described above the following symbolic reasoning can be developed: <ul style="list-style-type: none"> ○ If we increase the side length of a right-angled triangle (having height h and base line b) by a factor k, then the new height will be $k \cdot h$ and the base line $k \cdot b$. This means that the area is increased by a factor k^2, since $A_2 = \frac{1}{2} \cdot kh \cdot kb = k^2 \cdot A_1$, where A_1 is the area of the initial triangle. ○ For the general triangle with height h and baseline b one needs to argue why the height increases to $k \cdot h$. This could be
---	--

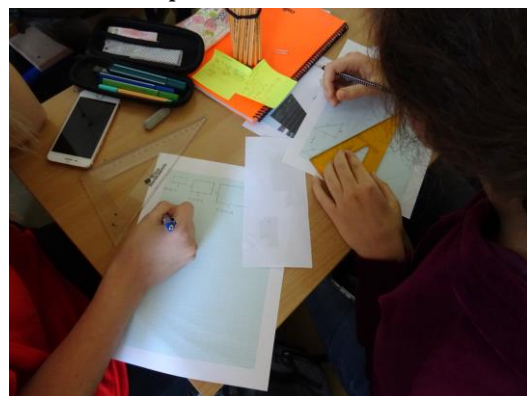


	<p>done by looking at the two right triangles which make up the general triangle.</p> <ul style="list-style-type: none"> ○ If we increase the side length of an arbitrary triangle (having two sides a, b and intermediate angle C) by a factor k, then the initial area can be calculated as $A_1 = \frac{1}{2} \cdot a \cdot b \cdot \sin(C)$. The increased area is then $A_2 = \frac{1}{2} \cdot ka \cdot kb \cdot \sin(C) = k^2 \cdot A_1$ ○ If we look at equal sided triangles then they can use the formula $A = \frac{\sqrt{3}}{4} s^2$, so if s increases by k, the area increases by a factor k^2 ○ For the right hand side picture (and other polygons) divide the polygon into triangles and calculate the sum of areas, using some of the above mentioned methods for triangles. ○ They can use Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$. This requires students to be able to conduct algebraic manipulations of expressions containing roots and powers.
--	--

Explanation of materials

The two pictures should engage the students in the devolution phase. The students are familiar with the idea of zooming in and out with pictures on computers and mobile phones. The chosen pictures show shapes which can be characterized as a triangle and a trapezoid. The latter can be covered by triangles and reduced to a question of enlargement of triangles. However, it requires an argument. This must be addressed by the teacher in the last devolution if students have not mentioned it already. In Appendix A you find a handout with three different sized versions of the pictures, which can be used to illustrate the problem to the students.

If the students and the teacher are accustomed to the use of ICT in the classroom, the picture can be integrated in the ICT (e.g. Geogebra) and an electronic file shared with the students. One might consider to mark the points of the vertices, if the syntax of calculating the area should be incorporated in the electronic worksheet, or if the point of the vertices should be movable. Appendices B, C and D all contain examples of worksheets in Geogebra and Geometer Sketchpad, where students can experiment with areas of different polygons.



Students working with grid paper and triangles (Croatia)

If the students are only familiar with very basic notions of shapes and their measures it might be needed to let them start drawing shapes on grid paper and count the numbers of covered squares.



In order to enlarge shapes and still preserve the angles, the students might need to be able to cut the shapes and use them when drawing the enlarged shapes. Hence, the students will need scissors and rulers as well. The picture shows students who created and enlarged their own shapes on grid paper.

If the students are more comfortable working with concrete models of polygons (for example triangles) different models should be provided. They enable students to get an idea of enlargement of area with a factor which is a natural number and help them transit to iconic and symbolic presentation of the problem.

A full report on the realisation of the scenario can be found in Appendix E.

Variations based on didactic variables

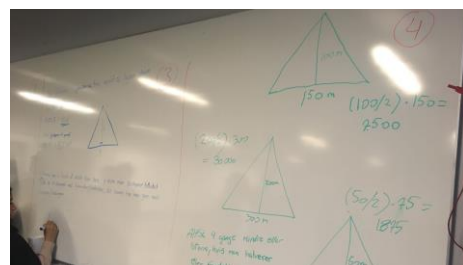
The focus in the didactical phases should be on students' formulations and the validation of them. Solutions should not be hinted at in didactic phases. In this section, we discuss what could be changed above (didactic variables).

The milieu: the pictures could be chosen differently, but should still contain clear shapes of a triangle and a quadrilateral (not a rectangle). Students could be encouraged to use ICT, but if designing a dynamic spreadsheet with the pictures incorporated, make sure it also requires work with formulae. They might then reach the conclusion that the area is enlarged by factor k^2 , but might not be able to develop an algebraic argument. This may then be done in a follow-up lesson. If designing a worksheet, avoid "hints" beyond the problem (e.g. fill-in tables).

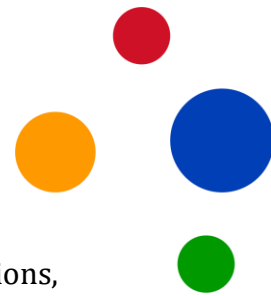
During the validation, it is important that any wrong strategies or formulas are corrected – as far as possible, by other students. The teacher can engage the rest of the class with questions to certain students like: Can you repeat what was just said? Is it correct? Why do you think so? Where do you know this from? Does it apply to both figures? To all shapes? What is the relation between $A = \frac{1}{2} \cdot h \cdot b$ and $A = \frac{1}{2} \cdot a \cdot b \cdot \sin(C)$? etc. What questions are asked depends on the prerequisites and achievements of the class.

The *length* of the phases could certainly be adapted to students work, but not prolonged much.

During the first two didactical phases, students should not be told what to discover or be reminded of what a scale factor of length is. If the teacher is in doubt whether the students have the prerequisites specified above, the teacher should pose questions such as: When are the shapes similar? How can we compare them? Is it reasonable to compare them? etc. The proposed questions should only be posed to groups or individuals, if most other students seem to have the



Student group's solutions on the white board
(Denmark)



required knowledge. If the majority of the class needs to consider those questions, the phase should be shortened and they should be posed in plenum. The teacher should not lecture each group separately. Further, it is *not needed* to stay with the group until they reach an answer to such a question. Regard it as a minor devolution of a limited problem and let the students act, formulate and validate. Do not support with further questions or hint the answer.

Interventions during *third phases of action*, formulation and validation: The main ideas are similar to those above. If some groups find it very difficult to get started, the teacher can suggest that they compare their strategy with a certain one from another group. This comparable strategy should be well chosen mathematically, meaning that there are some clear relations. This is like devolving a new slightly less open sub-problem to the group.

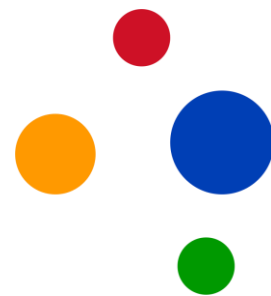
During *the final institutionalization*: it is important that most (if not all) strategies shared in the class are addressed and linked to one another. For instance, if one group uses the formula $A = \frac{1}{2}hb$ and another $A = \frac{1}{2}ab \sin(C)$, it should be argued (hopefully by some student group) that $h = a \cdot \sin(C)$. Such mathematical arguments should be formulated in detail (written down) by the teacher before the module begins, as a preparation of the teaching. Considering all possible strategies helps the teacher navigate and foresee the students' inquiry processes. While teaching, do remember that you interact with a dynamic system – students should be allowed to adapt to the milieu, so you cannot expect them all to deliver the same answer!

The inquiry process involves *all* phases. It may take more than one session for them to freely engage in this kind of teaching.

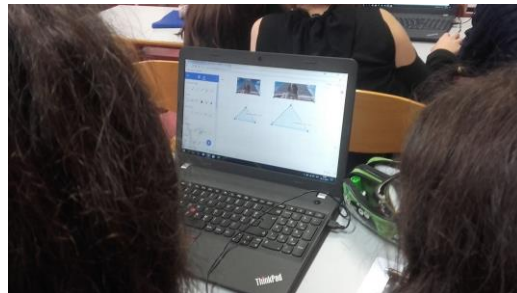
Some teachers create a schema of possible student strategies, for use during didactical phases. The expected strategies can be listed on a piece of paper and for each strategy the teacher formulates e.g. 3 questions that might be fruitful for the teacher to pose when a group presents a given strategy. During the didactical phases the teacher can note what groups discuss the different strategies and use this to organize the subsequent didactic formulation.

Observations from practice

The most important observation from the scenario is that some teachers tend to teach the students during all phases of the scenario. This ruins the didactic potential of the situation and strongly distracts the students. It was also observed that some students worked with the idea of rate of increase or decrease (what percentage does the area increase if the side lengths are increased by 30%). This can be related to scale factor of length letting $k = 1.30$. Then, during the last formulation and validation phase, one can ask students to relate their results to the scale factor. Groups choosing Heron's formula as a strategy might have a special opportunity to recall (or be reminded of) calculation rules for powers, roots and parentheses.



If the teacher invites the students to work with ICT, (s)he needs to prepare questions regarding reaching a mathematical argument e.g. drawing on algebraic manipulations. This can be done by posing questions, such as: What is the computer doing when it measures the area? How can the computer find the needed information/the side lengths, when point of vertexes are given? What formula do you think the computer might use and how does it find with the numbers it provides? When the computer increases the picture by 25%, what is it that becomes 25% larger?



Students investigate triangle measures in Geogebra (Slovenia).

If the teacher chooses other shapes (as some do) it should be carefully considered if the pictures change the problem. If the pictures clearly show a third dimension it might confuse some students. If the vertexes are not shown, it is difficult to determine the side lengths. If you choose shapes that are more complicated, it might confuse some students, and engage others. Hence, the choice should depend on the teacher's knowledge about the class. Non-polygonal shapes should be avoided in this situation.

Evaluation tools

At the end of the lesson or soon after, the following tasks can be used for a quick test of the knowledge students developed during the scenario:

- 1) Your friend says that to construct a square which is half as big (in area) as a given square, you must simply halve the side length. What do you think?
Answer: False. If you halve the length, the area is reduced to a quarter.
- 2) A triangle T has sides of length 3cm, 5cm and 7cm. Triangle T^* has the same angles but four times the area of T . What are the side lengths of T^* ?
Answer: To get 4 times the area, sides are doubled: 6 cm, 10 cm, 14 cm.
- 3) A figure, such as a triangle, is drawn on A4 paper. Using a copy machine it is magnified to A3 (so as you know, area gets doubled). What happens to the side lengths of the figure?
Answer: as areas get doubled, the area factor is $k^2 = 2$, where k is the scale factor. Thus, the side lengths are multiplied by $k = \sqrt{2}$.

Suggestion for further problems regarding area and enlargement

Tools: Paper, pen, ruler, protractor, calculator or similar.

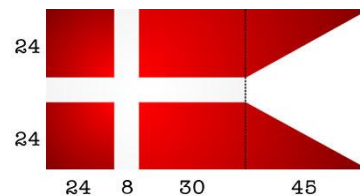
Solutions must include thorough explanations of how results are obtained.

1. The paper format A5 is a downsizing of the paper format A4 obtained by folding the A4 format along its middle line. The long side of an A4 sheet measures 297 mm. What is the length of the short side?

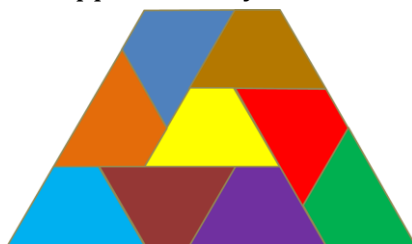


- Gold-leaves sell at the price of 27,99 \$ for 100 sheets of 2,2" x 2,0". An artist wants to gold-cover a letter A on a billboard measuring 3,5 m x 8 m. She is told that this will be too expensive and is asked to downsize the billboard to lower the gold cost by 30 %. What are the dimensions of the downsized billboard?
- On a city map of scale 1:7200 a park is estimated to have the area 12 cm². Give an estimate of the actual area of the park.

- Regulations for the Danish swallow-tailed flag stipulate the dimensions shown in the picture. It is recommended that the width of the flag is 1/5 of the height of the flag pole. Thus, if the unit of length is 1 cm, the flag shown is recommended for a pole of height 280 cm. What is the area of a swallow-tailed flag recommended for a pole of height 1120 cm?



- In 1854 the diameter of a one-dollar gold coin was changed from 13 mm to 15 mm without changing its weight. By what percent was the area increased? By what percent was the thickness decreased?
- The X-mas decorations to the right are made by forming sectors of circles into cones. For the small cones one uses approximately 350 cm² of carton. Give an estimate on how much carton is



- needed for the big cone.
- A fourth grader learning about triangles made the art picture shown to the left. The width of the shown picture is 5 cm. In the original picture, the area of the little yellow triangle in the middle is 10 cm². What is the width of the original picture?
- What conclusions about enlargement and scaling can you extract from the triangle art picture to the right?

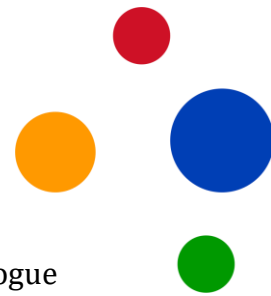


Rationale and RME perspectives on the scenario

Relevance, applicability:

We consider the following three perspectives:

- Real life: This knowledge is related to everyday experience with enlargement of photos or documents. Students need to be aware of the various interpretations of "two times as large as the original photo". Or ordering two times as much as 1 m³ of sand results in a cube of: ? m × ? m × ? m of sand.
- World of work: Being able to reason about enlargement and effects of scaling with a certain factor is relevant in many workplaces, for instance healthcare (% solutions) and architects (scale models).
- Further study: The knowledge and skills related to this topic can be used in many science disciplines (e.g. biology of animal-bone-shapes and physics of object-cooling). A historico-philosophical note for humanities-oriented



students: doubling the area of a square appears in Plato's famous dialogue *Menon*.

Inquiry skills

In the Area scenario students experience the importance of inquiry skills like formulating hypotheses, generating examples, experimenting systematically, organizing data, finding and justifying a formula, collaborating and communicating results. The extent by which these skills are explicitly addressed is largely dependent on the way by which the teacher involves them in feedback on the methods during the second validation phase when (s)he calls the groups to present. Furthermore, they can be part of the following formulation and institutionalization phases. In those cases, we suggest teachers to write them somewhere so that they can be returned to during follow up lessons.

Potential for a sequence of lessons:

The Area scenario could be a part of a larger series of lessons on enlargement and its relation to the change of lengths, areas and volumes of geometrical shapes.

- **Pre-knowledge**

For such a chapter, we expect students to be familiar with formulae for calculating the area and the volume of basic mathematical shapes like triangles, rectangles, circles, cubes and cylinders. Furthermore, they are familiar with the notion of a scale factor and its relation to enlarging shapes such that proportions between lengths remain the same (similarity).

- **An introduction: a context with a rich open problem, overarching the module. A problem in a context that naturally connects enlargement to change of lengths, areas and volumes and evokes the question of how they relate.**

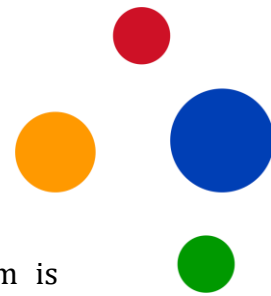
Possibilities:

- Animals – area/volume; dinosaurs
- Mini mundus – scale down by a factor – use LEGO-blocks
- Packing industry – material & volume & unit boxes
- Babushka – puppets (e.g. investigate length & weight)
- Analyze (Barbie) dolls and proportions when you enlarge

- ***Horizontal mathematising:* mathematical language is introduced to discuss the situation. The learners form a first informal model of the situation.**

Students explore relations by generating examples and measuring and try to discover what is going in their working center. They prepare a presentation of their findings.

The teacher guides the discussion on similarities and differences between these findings to reach the conclusion that different dimensions behave differently. This results in the need to investigate one of these dimensions in further detail. Let's take area.



- *Vertical mathematizing*: the mathematics involved in the problem is further developed. The model is made more abstract, more general.

Introduce Area problem by the Area scenario. This scenario is introduced in the context of enlarging two pictures, one of a pyramid and one of a building. This context (enlarging pictures) naturally leads to questions about changing lengths and areas in the picture. However, you might run the risk of students also involving characteristics of the scenes in the picture (e.g. what happens with the dimensions of a pyramid when you enlarge it?) and being distracted by aspects like the consequences of perspective projection.

This scenario results in an understanding of areas changing by a factor k^2 when lengths are scaled by factor k . Furthermore, the inquiry skills of generating examples, organizing your data and finding and justifying a formula can be in the focus of the formulation and the institutionalisation phases.

- *Conclusion and reflection and suggestions for further study*: the learner reflects, integrates ideas, makes concepts and skills explicit, the teacher highlights main learning points.

A follow up lesson could further investigate what the conclusion(s) of the scenario tell us about the initial findings in working centers: can we use k^2 , and inquiry strategies in those contexts? One or more of the initial problems can be investigated in more detail to come to similar conclusions about length and volume. And in the end a kind of understanding that scaling of lengths by factor k results in scaling of objects (e.g. area or volume) with dimension d (e.g. with $d = 2$ or $d = 3$) by factor k^d .